6-1 The Polygon Angle-Sum Theorems

Objectives To find the sum of the measures of the interior angles of a polygon
To find the sum of the measures of the exterior angles of a polygon

You don't need to draw all the diagonals from every vertex.

Getting Ready!

Sketch a convex pentagon, hexagon, and heptagon. For each figure, draw all the diagonals you can from one vertex. What conjecture can you make about the relationship between the number of sides of a polygon and the number of triangles formed by the diagonals from one vertex?

Dynamic Activity
Polygon Angle-Sum Theorem

The Solve It is related to a formula for the sum of the interior angle measures of a polygon. (In this textbook, a polygon is convex unless otherwise stated.)

Essential Understanding The sum of the interior angle measures of a polygon depends on the number of sides the polygon has.

By dividing a polygon with n sides into $(n - 2)$ triangles, you can show that the sum of the interior angle measures of any polygon is a multiple of 180.

Lesson Vocabulary
- equilateral polygon
- equiangular polygon
- regular polygon

take note

Theorem 6-1 Polygon Angle-Sum Theorem

The sum of the measures of the interior angles of an n-gon is $(n - 2)180$.

Problem 1 Finding a Polygon Angle Sum

What is the sum of the interior angle measures of a heptagon?

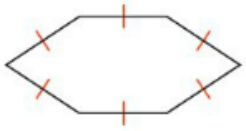
Think
How many sides does a heptagon have?
A heptagon has 7 sides.

$$\begin{aligned} \text{Sum} &= (n - 2)180 && \text{Polygon Angle-Sum Theorem} \\ &= (7 - 2)180 && \text{Substitute 7 for } n. \\ &= 5 \cdot 180 && \text{Simplify.} \\ &= 900 \end{aligned}$$

The sum of the interior angle measures of a heptagon is 900.

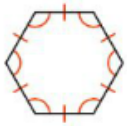
Got It? 1. **a.** What is the sum of the interior angle measures of a 17-gon?
b. Reasoning The sum of the interior angle measures of a polygon is 1980. How can you find the number of sides in the polygon?

An **equilateral polygon** is a polygon with all sides congruent.



An **equiangular polygon** is a polygon with all angles congruent.

A **regular polygon** is a polygon that is both equilateral and equiangular.



take note

Corollary to the Polygon Angle-Sum Theorem

The measure of each interior angle of a regular n-gon is $\frac{(n-2)180}{n}$.

You will prove the Corollary to the Polygon Angle-Sum Theorem in Exercise 43.

Problem 2 Using the Polygon Angle-Sum Theorem

Biology The common housefly, *Musca domestica*, has eyes that consist of approximately 4000 facets. Each facet is a regular hexagon. What is the measure of each interior angle in one hexagonal facet?

Think

How does the word *regular* help you answer the question?
The word *regular* tells you that each angle has the same measure.

$$\begin{aligned}
\text{Measure of an angle} &= \frac{(n-2)180}{n} && \text{Corollary to the Polygon Angle-Sum Theorem} \\
&= \frac{(6-2)180}{6} && \text{Substitute 6 for } n. \\
&= \frac{4 \cdot 180}{6} && \text{Simplify.} \\
&= 120
\end{aligned}$$

The measure of each interior angle in one hexagonal facet is 120.

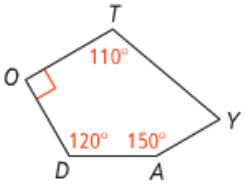
Got It? 2. What is the measure of each interior angle in a regular nonagon?

Problem 3 Using the Polygon Angle-Sum Theorem

Think

How does the diagram help you?
You know the number of sides and four of the five angle measures.

What is $m\angle Y$ in pentagon *TODAY*?



Use the Polygon Angle-Sum Theorem for $n = 5$.

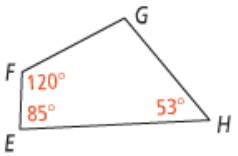
$$m\angle T + m\angle O + m\angle D + m\angle A + m\angle Y = (5 - 2)180$$

$$110 + 90 + 120 + 150 + m\angle Y = 3 \cdot 180 \quad \text{Substitute.}$$

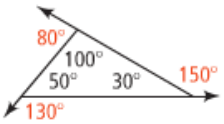
$$470 + m\angle Y = 540 \quad \text{Simplify.}$$

$$m\angle Y = 70 \quad \text{Subtract 470 from each side.}$$

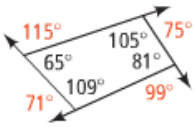
Got It? **3.** What is $m\angle G$ in quadrilateral *EFGH*?



You can draw exterior angles at any vertex of a polygon. The figures below show that the sum of the measures of the exterior angles, one at each vertex, is 360.



$80 + 150 + 130 = 360$



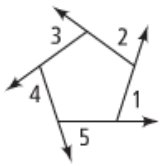
$115 + 75 + 99 + 71 = 360$

take note

Theorem 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

For the pentagon, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$.



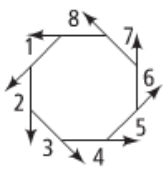
You will prove Theorem 6-2 in Exercise 39.

Problem 4 Finding an Exterior Angle Measure

Think

What kind of angle is $\angle 1$?
Looking at the diagram, you know that $\angle 1$ is an exterior angle.

What is $m\angle 1$ in the regular octagon at the right?



By the Polygon Exterior Angle-Sum Theorem, the sum of the exterior angle measures is 360. Since the octagon is regular, the interior angles are congruent. So their supplements, the exterior angles, are also congruent.

$$m\angle 1 = \frac{360}{8} \quad \text{Divide 360 by 8, the number of sides in an octagon.}$$

$$= 45 \quad \text{Simplify.}$$

Got It? **4.** What is the measure of an exterior angle of a regular nonagon?

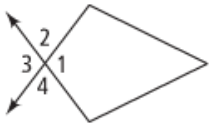
Lesson Check

Do you know HOW?

1. What is the sum of the interior angle measures of an 11-gon?
2. What is the sum of the measures of the exterior angles of a 15-gon?
3. Find the measures of an interior angle and an exterior angle of a regular decagon.

Do you UNDERSTAND?

4. **Vocabulary** Can you draw an equiangular polygon that is not equilateral? Explain.
5. **Reasoning** Which angles are the exterior angles for $\angle 1$? What do you know about their measures? Explain.

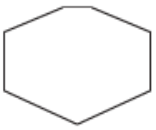


6. **Error Analysis** Your friend says that she measured an interior angle of a regular polygon as 130. Explain why this result is impossible.

Practice and Problem-Solving Exercises

A Practice

Find the sum of the interior angle measures of each polygon. See Problem 1.

7. 

8. 35-gon
9. 14-gon
10. 20-gon
11. 1002-gon

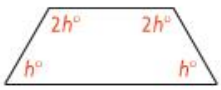
Find the measure of one interior angle in each regular polygon. See Problem 2.

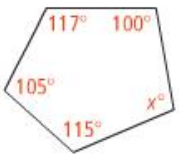
12.

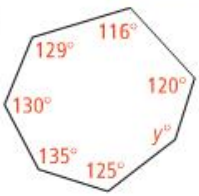
13.

14.

Algebra **Find the missing angle measures.** See Problem 3.

15. 

16. 

17. 

Find the measure of an exterior angle of each regular polygon. See Problem 4.

18. pentagon
19. 36-gon
20. 18-gon
21. 100-gon

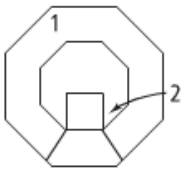
B Apply

The sum of the interior angle measures of a polygon with n sides is given. Find n.

22. 180
23. 1080
24. 1980
25. 2880

26. Open-Ended Sketch an equilateral polygon that is not equiangular.

27. Stage Design A theater-in-the-round allows for a play to have an audience on all sides. The diagram at the right shows a platform constructed for a theater-in-the-round stage. What type of regular polygon is the largest platform? Find the measure of each numbered angle.

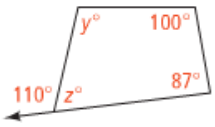


28. Think About a Plan A triangle has two congruent interior angles and an exterior angle that measures 100. Find two possible sets of interior angle measures for the triangle.

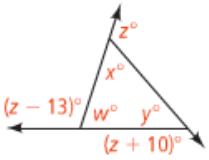
- How can a diagram help you?
- What is the sum of the angle measures in a triangle?

Algebra **Find the value of each variable.**

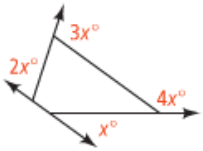
29.



30.



31.



The measure of an exterior angle of a regular polygon is given. Find the measure of an interior angle. Then find the number of sides.

32. 72 **33.** 36 **34.** 18 **35.** 30 **36.** x

Packaging **The gift package at the right contains fruit and cheese. The fruit is in a container that has the shape of a regular octagon. The fruit container fits in a square box. A triangular cheese wedge fills each corner of the box.**

37. Find the measure of each interior angle of a cheese wedge.

38. Reasoning Show how to rearrange the four pieces of cheese to make a regular polygon. What is the measure of each interior angle of the polygon?

39. Algebra A polygon has n sides. An interior angle of the polygon and an adjacent exterior angle form a straight angle.

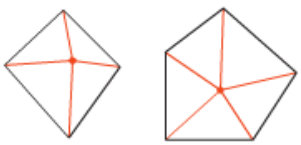
a. What is the sum of the measures of the n straight angles?

b. What is the sum of the measures of the n interior angles?

c. Using your answers above, what is the sum of the measures of the n exterior angles?

d. What theorem do the steps above prove?

40. Reasoning Your friend says she has another way to find the sum of the interior angle measures of a polygon. She picks a point inside the polygon, draws a segment to each vertex, and counts the number of triangles. She multiplies the total by 180, and then subtracts 360 from the product. Does her method work? Explain.



41. Algebra The measure of an interior angle of a regular polygon is three times the measure of an exterior angle of the same polygon. What is the name of the polygon?

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Challenge

42. Probability Find the probability that the measure of an interior angle of a regular n-gon is a positive integer when n is an integer and $3 \leq n \leq 12$.

43. a. In the Corollary to the Polygon Angle-Sum Theorem, explain why the measure of an interior angle of a regular n-gon is given by the formulas $\frac{180(n-2)}{n}$ and $180 - \frac{360}{n}$.

b. Use the second formula to explain what happens to the measures of the interior angles of regular n-gons as n becomes a large number. Explain also what happens to the polygons.

44. $ABCDEFGHJK$ is a regular decagon. A ray bisects $\angle C$, and another ray bisects $\angle D$. The two rays intersect in the decagon's interior. Find the measure of the acute angles formed by the intersecting rays.

Standardized Test Prep

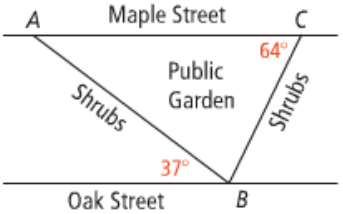
GRIDDED RESPONSE

SAT/ACT

45. The car at each vertex of a Ferris wheel holds a maximum of five people. The sum of the interior angle measures of the Ferris wheel is 7740. What is the maximum number of people the Ferris wheel can hold?

46. A rectangle and a square have equal areas. The rectangle has length 9 cm and width 4 cm. What is the perimeter of the square, in centimeters?

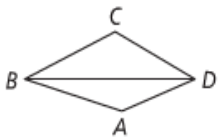
47. The Public Garden is located between two parallel streets: Maple Street and Oak Street. The garden faces Maple Street and is bordered by rows of shrubs that intersect Oak Street at point B. What is $m\angle ABC$, the angle formed by the shrubs?



48. $\triangle ABC \cong \triangle DEF$. If $m\angle A = 3x + 4$, $m\angle C = 2x$, and $m\angle E = 4x + 5$, what is $m\angle B$?

Mixed Review

49. If $\overline{AB} \cong \overline{CB}$ and $m\angle ABD < m\angle CBD$, which is longer, $\overline{AD}$ or $\overline{CD}$? Explain.



See Lesson 5-7.

Name the property that justifies each statement.

See Lesson 2-5.

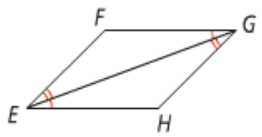
50. $4(2a - 3) = 8a - 12$ **51.** $\overline{RS} \cong \overline{RS}$ **52.** If $\angle 1 \cong \angle 4$, then $\angle 4 \cong \angle 1$.

Get Ready! **To prepare for Lesson 6-2, do Exercises 53 and 54.**

Use the figure below.

See Lessons 4-1 and 4-3.

53. Name the postulate or theorem that justifies $\triangle EFG \cong \triangle GHE$.



54. Complete each statement.

a. $\angle FEG \cong$ ■ **b.** $\angle EFG \cong$ ■ **c.** $\angle FGE \cong$ ■

d. $\overline{EF} \cong$ ■ **e.** $\overline{FG} \cong$ ■ **f.** $\overline{GE} \cong$ ■

6-2 Properties of Parallelograms

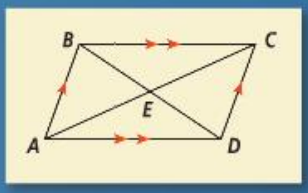
Objectives To use relationships among sides and angles of parallelograms
To use relationships among diagonals of parallelograms

Don't settle for your answer too soon! Be sure to find all pairs of congruent triangles.

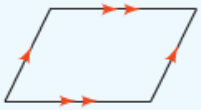
SOLVE IT!

Getting Ready!

Use the information given in the diagram. Which triangles are congruent? How do you know?



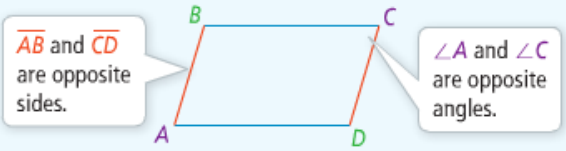
A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In the Solve It, you made some conjectures about the characteristics of a parallelogram. In this lesson, you will verify whether your conjectures are correct.



Lesson Vocabulary
- parallelogram
- opposite sides
- opposite angles
- consecutive angles

Essential Understanding Parallelograms have special properties regarding their sides, angles, and diagonals.

In a quadrilateral, **opposite sides** do not share a vertex and **opposite angles** do not share a side.



You can abbreviate *parallelogram* with the symbol $\square$ and *parallelograms* with the symbol $\square\!\!\square$. You can use what you know about parallel lines and transversals to prove some theorems about parallelograms.

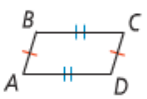
take note

Theorem 6-3

Theorem
If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If . . .
$ABCD$ is a $\square$

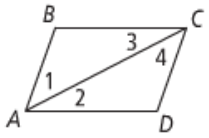
Then . . .
$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



Proof Proof of Theorem 6-3

Given: $\square ABCD$

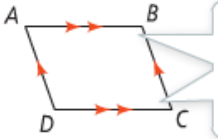
Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



Statements	Reasons
1) $ABCD$ is a parallelogram.	1) Given
2) $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$	2) Definition of parallelogram
3) $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 2$	3) If lines are $\parallel$, then alt. int. $\measuredangle$ are $\cong$.
4) $\overline{AC} \cong \overline{AC}$	4) Reflexive Property of $\cong$
5) $\triangle ABC \cong \triangle CDA$	5) ASA
6) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$	6) Corresp. parts of $\cong$ $\measuredangle$ are $\cong$.

Angles of a polygon that share a side are **consecutive angles.** In the diagram, $\angle A$ and $\angle B$ are consecutive angles because they share side $\overline{AB}$.

The theorem below uses the fact that consecutive angles of a parallelogram are same-side interior angles of parallel lines.



take note

Theorem 6-4

Theorem
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If . . .
$ABCD$ is a $\square$

Then . . .

$m\angle A + m\angle B = 180$
$m\angle B + m\angle C = 180$
$m\angle C + m\angle D = 180$
$m\angle D + m\angle A = 180$

You will prove Theorem 6-4 in Exercise 32.

ONLINE PROBLEMS

Problem 1 Using Consecutive Angles

Plan

What information from the diagram helps you get started?
From the diagram, you know $m\angle PSR$ and that $\angle P$ and $\angle PSR$ are consecutive angles. So, you can *write an equation* and solve for $m\angle P$.

Multiple Choice What is $m\angle P$ in $\square PQRS$?

(A) 26 (C) 116
(B) 64 (D) 126

$m\angle P + m\angle S = 180$	Consecutive angles of a $\square$ are suppl.
$m\angle P + 64 = 180$	Substitute.
$m\angle P = 116$	Subtract 64 from each side.

The correct answer is C.

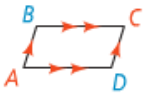
Got It? 1. Suppose you adjust the lamp so that $m\angle S = 86$. What is $m\angle R$ in $\square PQRS$?

take note

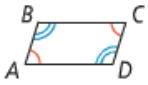
Theorem 6-5

Theorem
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If . . .
$ABCD$ is a $\square$.



Then . . .
$\angle A \cong \angle C$ and $\angle B \cong \angle D$



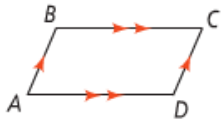
A proof of Theorem 6-5 in Problem 2 uses the consecutive angles of a parallelogram and the fact that supplements of the same angle are congruent.

Proof

Problem 2 Using Properties of Parallelograms in a Proof

Given: $\square ABCD$

Prove: $\angle A \cong \angle C$ and $\angle B \cong \angle D$



$ABCD$ is a $\square$.
Given

$\angle A$ and $\angle B$ are consecutive $\measuredangle$.
Def. of consecutive $\measuredangle$

$\angle B$ and $\angle C$ are consecutive $\measuredangle$.
Def. of consecutive $\measuredangle$

$\angle C$ and $\angle D$ are consecutive $\measuredangle$.
Def. of consecutive $\measuredangle$

$\angle A$ and $\angle B$ are supplementary.
Consecutive $\measuredangle$ are supplementary.

$\angle B$ and $\angle C$ are supplementary.
Consecutive $\measuredangle$ are supplementary.

$\angle C$ and $\angle D$ are supplementary.
Consecutive $\measuredangle$ are supplementary.

$\angle A \cong \angle C$
Supplements of the same $\angle$ are $\cong$.

$\angle B \cong \angle D$
Supplements of the same $\angle$ are $\cong$.

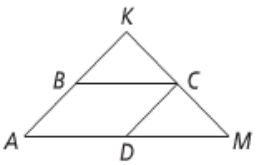
Think

Why is a flow proof useful here?
A flow proof allows you to see how the pairing of two statements leads to a conclusion.

Got It? **2.** Use the diagram at the right.

Given: $\square ABCD$, $\overline{AK} \cong \overline{MK}$

Prove: $\angle BCD \cong \angle CMD$



The diagonals of parallelograms have a special property.

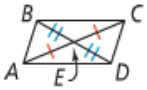
take note

Theorem 6-6

Theorem
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If . . .
$ABCD$ is a $\square$

Then . . .
$\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$

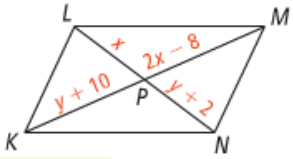


You will prove Theorem 6-6 in Exercise 13.

You can use Theorem 6-6 to find unknown lengths in parallelograms.

Problem 3 Using Algebra to Find Lengths

Solve a system of linear equations to find the values of x and y in $\square KLMN$. What are KM and LN?



Think

The diagonals of a parallelogram bisect each other.

Write

$\overline{KP} \cong \overline{MP}$
$\overline{LP} \cong \overline{NP}$

Set up a system of linear equations by substituting the algebraic expressions for each segment length.

① $y + 10 = 2x - 8$
② $x = y + 2$

Substitute $(y + 2)$ for x in equation ①. Then solve for y.

$y + 10 = 2(y + 2) - 8$
$y + 10 = 2y + 4 - 8$
$y + 10 = 2y - 4$
$10 = y - 4$
$14 = y$

Substitute 14 for y in equation ②. Then solve for x.

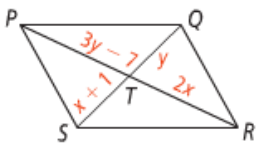
$x = 14 + 2$
$= 16$

Use the values of x and y to find KM and LN.

$KM = 2(KP)$
$= 2(y + 10)$
$= 2(14 + 10)$
$= 48$

$LN = 2(LP)$
$= 2(x)$
$= 2(16)$
$= 32$

Got It? **3. a.** Find the values of x and y in $\square PQRS$ at the right. What are PR and SQ?



b. Reasoning In Problem 3, does it matter which variable you solve for first? Explain.

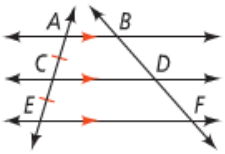
You will use parallelograms to prove the following theorem.

take note

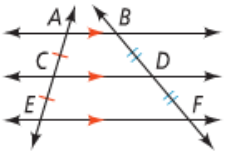
Theorem 6-7

Theorem
If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

If . . .
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overline{AC} \cong \overline{CE}$



Then . . .
$\overline{BD} \cong \overline{DF}$



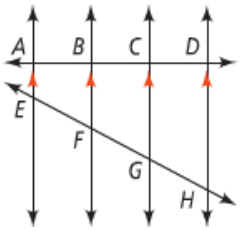
You will prove Theorem 6-7 in Exercise 43.

Problem 4 Using Parallel Lines and Transversals

Plan

What information do you need?
You know the length of $\overline{EF}$. To find EH, you need the lengths of $\overline{FG}$ and $\overline{GH}$.

In the figure, $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CG} \parallel \overleftrightarrow{DH}$, $AB = BC = CD = 2$, and $EF = 2.25$. What is EH?



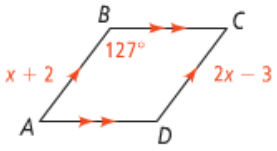
$EF = FG = GH$	Since $\parallel$ lines divide $\overline{AD}$ into equal parts, they also divide $\overline{EH}$ into equal parts.
$EH = EF + FG + GH$	Segment Addition Postulate
$EH = 2.25 + 2.25 + 2.25 = 6.75$	Substitute.

Got It? **4.** Use the figure in Problem 4. If $EF = FG = GH = 6$ and $AD = 15$, what is CD?

Lesson Check

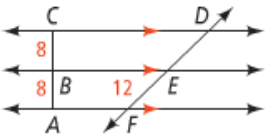
Do you know HOW?

Use the diagram of $\square ABCD$ to find each value.



1. $m\angle A$
2. $m\angle D$
3. x
4. AB

5. What are ED and FD in the figure at the right?

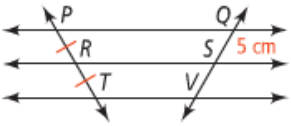


Do you UNDERSTAND?

6. **Reasoning** If you know one angle measure of a parallelogram, how do you find the other three angle measures? Explain.

7. **Compare and Contrast** What is the difference between a quadrilateral and a parallelogram?

8. **Error Analysis** Your classmate says that $QV = 10$. Explain why the statement may not be correct.

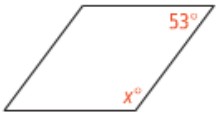


Practice and Problem-Solving Exercises

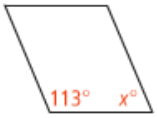
A Practice

Algebra Find the value of x in each parallelogram. See Problem 1.

9.

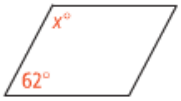


10.



11.

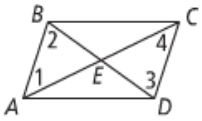
12.



13. Developing Proof Complete this two-column proof of Theorem 6-6. See Problem 2.

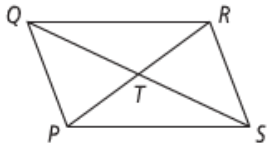
Given: $\square ABCD$

Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.



Statements	Reasons
1) $ABCD$ is a parallelogram.	1) Given
2) $\overline{AB} \parallel \overline{DC}$	2) a. ?
3) $\angle 1 \cong \angle 4; \angle 2 \cong \angle 3$	3) b. ?
4) $\overline{AB} \cong \overline{DC}$	4) c. ?
5) d. ?	5) ASA
6) $\overline{AE} \cong \overline{CE}; \overline{BE} \cong \overline{DE}$	6) e. ?
7) f. ?	7) Definition of bisector

Algebra Find the values of x and y in $\square PQRS$. See Problem 3.

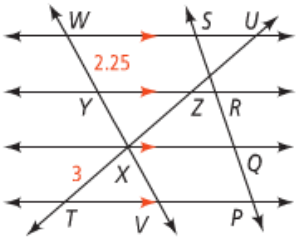


14. $PT = 2x, TR = y + 4, QT = x + 2, TS = y$

15. $PT = x + 2, TR = y, QT = 2x, TS = y + 3$

16. $PT = y, TR = x + 3, QT = 2y, TS = 3x - 1$

In the figure, $PQ = QR = RS$. Find each length. See Problem 4.



17. ZU

18. XZ

19. TU

20. XV

21. YX

22. YV

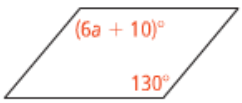
23. WX

24. WV

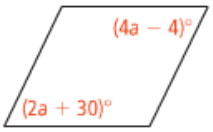
B Apply

Algebra Find the value(s) of the variable(s) in each parallelogram.

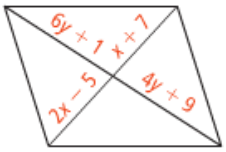
25.



26.



27.

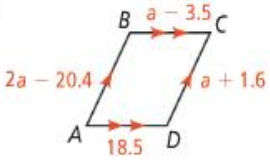


28. Think About a Plan What are the values of x and y in the parallelogram?

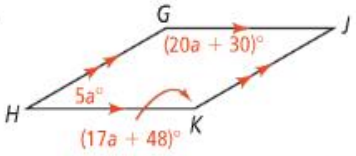
- How are the angles related?
- Which variable should you solve for first?

Algebra **Find the value of a. Then find each side length or angle measure.**

29.



30.



31. Studio Lighting A pantograph is an expandable device shown at the right. Pantographs are used in the television industry in positioning lighting and other equipment. In the photo, points D, E, F, and G are the vertices of a parallelogram. $\square DEFG$ is one of many parallelograms that change shape as the pantograph extends and retracts.

a. If $DE = 2.5$ ft, what is FG? **b.** If $m\angle E = 129$, what is $m\angle G$?

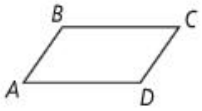
c. What happens to $m\angle D$ as $m\angle E$ increases or decreases? Explain.

32. Prove Theorem 6-4.

Proof

Given: $\square ABCD$

Prove: $\angle A$ is supplementary to $\angle B$.
$\angle A$ is supplementary to $\angle D$.



Use the diagram at the right for each proof.

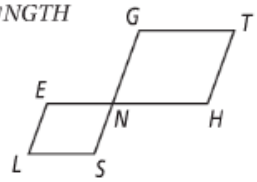
Proof

33. Given: $\square LENS$ and $\square NGTH$

Prove: $\angle L \cong \angle T$

34. Given: $\square LENS$ and $\square NGTH$

Prove: $\overline{LS} \parallel \overline{GT}$



35. Given: $\square LENS$ and $\square NGTH$

Prove: $\angle E$ is supplementary to $\angle T$.

Use the diagram at the right for each proof.

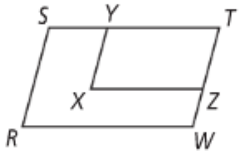
Proof

36. Given: $\square RSTW$ and $\square XYTZ$

Prove: $\angle R \cong \angle X$

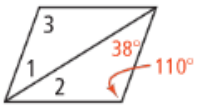
37. Given: $\square RSTW$ and $\square XYTZ$

Prove: $\overline{XY} \parallel \overline{RS}$

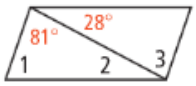


Find the measures of the numbered angles for each parallelogram.

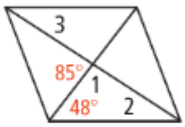
38.



39.



40.



41. Algebra The perimeter of $\square ABCD$ is 92 cm. AD is 7 cm more than twice AB. Find the lengths of all four sides of $\square ABCD$.

Challenge

42. Writing Is there an SSSS congruence theorem for parallelograms? Explain.

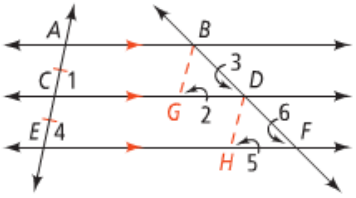
43. Prove Theorem 6-7. Use the diagram at the right.

Proof

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, $\overline{AC} \cong \overline{CE}$

Prove: $\overline{BD} \cong \overline{DF}$

(*Hint:* Draw lines through B and D parallel to $\overleftrightarrow{AE}$ and intersecting $\overleftrightarrow{CD}$ at G and $\overleftrightarrow{EF}$ at H.)

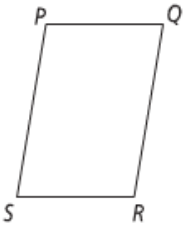


44. Measurement Explain how to separate a blank card into three strips that are the same height by using lined paper, a straightedge, and Theorem 6-7.

Standardized Test Prep

SAT/ACT

45. $PQRS$ is a parallelogram with $m\angle Q = 4x$ and $m\angle R = x + 10$. Which statement explains why you can use the equation $4x + (x + 10) = 180$ to solve for x?



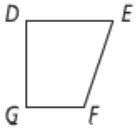
(A) The measures of the interior angles of a quadrilateral have a sum of 360.

(B) Opposite sides of a parallelogram are congruent.

(C) Opposite angles of a parallelogram are congruent.

(D) Consecutive angles of a parallelogram are supplementary.

46. In the figure of $DEFG$ at the right, $\overline{DE} \parallel \overline{GF}$. Which statement must be true?



(F) $m\angle D + m\angle E = 180$

(G) $m\angle D + m\angle G = 180$

(H) $\overline{DE} \cong \overline{GF}$

(I) $\overline{DG} \cong \overline{EF}$

47. An obtuse triangle has side lengths of 5 cm, 9 cm, and 12 cm. What is the length of the side opposite the obtuse angle?

(A) 5 cm (B) 9 cm (C) 12 cm (D) not enough information

Short Response

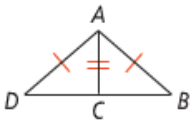
48. Find the measure of one exterior angle of a regular hexagon. Explain your method.

Mixed Review

Find the sum of the measures of the interior angles of each polygon. See Lesson 6-1.

49. decagon **50.** 16-gon **51.** 25-gon **52.** 40-gon

53. What additional information do you need to prove $\triangle ADC \cong \triangle ABC$ by the HL Theorem?



See Lesson 4-6.

Get Ready! **To prepare for Lesson 6-3, do Exercise 54.**

54. Two consecutive angles in a parallelogram have measures $x + 5$ and $4x - 10$. Find the measure of the smaller angle.

See Lesson 6-2.

6-3 Proving That a Quadrilateral Is a Parallelogram

Objective To determine whether a quadrilateral is a parallelogram

Can you find parallelograms in the pattern?

Getting Ready!

Each section of glass in the exterior of a building in Macau, China, forms an equilateral triangle. Do you think the window washer's feet stay parallel to the ground as he lands at each level of windows? Explain. (Assume that the bases of the lowest triangles are parallel to the ground.)

Dynamic Activity Parallelogram Conditions

In the Solve It, you used angle properties to show that lines are parallel. In this lesson, you will apply the same properties to show that a quadrilateral is a parallelogram.

Essential Understanding You can decide whether a quadrilateral is a parallelogram if its sides, angles, and diagonals have certain properties.

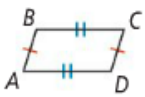
In Lesson 6-2, you learned theorems about the properties of parallelograms. In this lesson, you will learn the converses of those theorems. That is, if a quadrilateral has certain properties, then it must be a parallelogram. Theorem 6-8 is the converse of Theorem 6-3.

take note

Theorem 6-8

Theorem
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

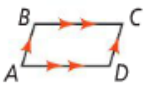
If . . .



$\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{DA}$

Then . . .

$ABCD$ is a $\square$



You will prove Theorem 6-8 in Exercise 20.

Theorems 6-9 and 6-10 are the converses of Theorems 6-4 and 6-5, respectively. They use angle relationships to conclude that a quadrilateral is a parallelogram.

take note

Theorem 6-9

Theorem
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.

If . . .

$m\angle A + m\angle B = 180$
$m\angle A + m\angle D = 180$

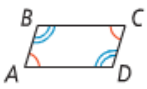
Then . . .
$ABCD$ is a ▱

You will prove Theorem 6-9 in Exercise 21.

Theorem 6-10

Theorem
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If . . .



$\angle A \cong \angle C$
$\angle B \cong \angle D$

Then . . .
$ABCD$ is a ▱

You will prove Theorem 6-10 in Exercise 18.

You can use algebra together with Theorems 6-8, 6-9, and 6-10 to find segment lengths and angle measures that assume that a quadrilateral is a parallelogram.

ROUTINE PROBLEMS

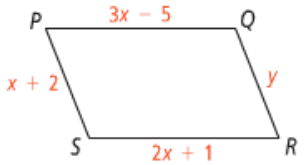
Problem 1 Finding Values for Parallelograms

GRIDDED RESPONSE

For what value of y must $PQRS$ be a parallelogram?

Plan

Which theorem should you use?
The diagram gives you information about sides. Use Theorem 6-8 because it uses sides to conclude that a quadrilateral is a parallelogram.



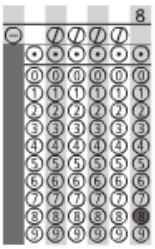
Step 1 Find x.

$3x - 5 = 2x + 1$	If opp. sides are ≅, then the quad. is a ▱.
$x - 5 = 1$	Subtract $2x$ from each side.
$x = 6$	Add 5 to each side.

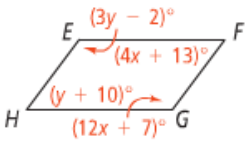
Step 2 Find y.

$y = x + 2$	If opp. sides are ≅, then the quad. is a ▱.
$= 6 + 2$	Substitute 6 for x.
$= 8$	Simplify.

For $PQRS$ to be a parallelogram, the value of y must be 8.



Got It? **1.** Use the diagram at the right. For what values of x and y must $EFGH$ be a parallelogram?



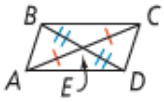
You know that the converses of Theorems 6-3, 6-4, and 6-5 are true. Using what you have learned, you can show that the converse of Theorem 6-6 is also true.

take note

Theorem 6-11

Theorem
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If . . .



$\overline{AE} \cong \overline{CE}$
$\overline{BE} \cong \overline{DE}$

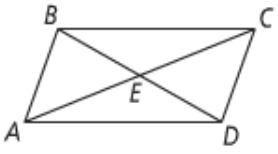
Then . . .
$ABCD$ is a $\square$

Proof

Proof of Theorem 6-11

Given: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.

Prove: $ABCD$ is a parallelogram.



$\overline{AC}$ and $\overline{BD}$ bisect each other at E.
Given

$\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$
Def. of segment bisector

$\angle AEB \cong \angle CED$
Vertical $\measuredangle$ are $\cong$.

$\angle BEC \cong \angle DEA$
Vertical $\measuredangle$ are $\cong$.

$\triangle AEB \cong \triangle CED$
SAS

$\triangle BEC \cong \triangle DEA$
SAS

$\angle BAE \cong \angle DCE$
Corresp. parts of $\cong$ $\measuredangle$ are $\cong$.

$\angle ECB \cong \angle EAD$
Corresp. parts of $\cong$ $\measuredangle$ are $\cong$.

$\overline{AB} \parallel \overline{CD}$
If alternate interior $\measuredangle \cong$, then lines are $\parallel$.

$\overline{BC} \parallel \overline{AD}$
If alternate interior $\measuredangle \cong$, then lines are $\parallel$.

$ABCD$ is a parallelogram.
Def. of parallelogram

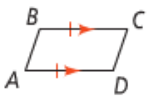
Theorem 6-12 suggests that if you keep two objects of the same length parallel, such as cross-country skis, then the quadrilateral formed by connecting their endpoints is always a parallelogram.

take note

Theorem 6-12

Theorem
If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

If . . .



$\overline{BC} \cong \overline{DA}$
$\overline{BC} \parallel \overline{DA}$

Then . . .
$ABCD$ is a $\square$

You will prove Theorem 6-12 in Exercise 19.

ONLINE PROBLEM SOLVING

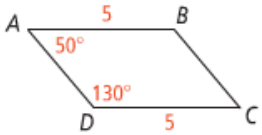
Problem 2 Deciding Whether a Quadrilateral Is a Parallelogram

Think
How do you decide if you have enough information?
If you can satisfy every condition of a theorem about parallelograms, then you have enough information.

Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

A **Given:** $AB = 5, CD = 5,$
$m\angle A = 50, m\angle D = 130$

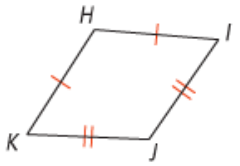
Prove: $ABCD$ is a parallelogram.



Yes. Same-side interior angles A and D are supplementary, so $\overline{AB} \parallel \overline{CD}$. Since $\overline{AB} \cong \overline{CD}$, $ABCD$ is a parallelogram by Theorem 6-12.

B **Given:** $\overline{HI} \cong \overline{HK}, \overline{JI} \cong \overline{JK}$

Prove: $HIJK$ is a parallelogram.

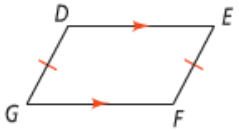


No. By Theorem 6-8, you need to show that both pairs of *opposite* sides are congruent, not consecutive sides.

Got It? **2.** Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

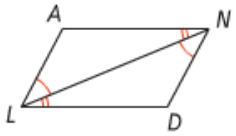
a. Given: $\overline{EF} \cong \overline{GD}, \overline{DE} \parallel \overline{FG}$

Prove: $DEFG$ is a parallelogram.



b. Given: $\angle ALN \cong \angle DNL, \angle ANL \cong \angle DLN$

Prove: $LAND$ is a parallelogram.

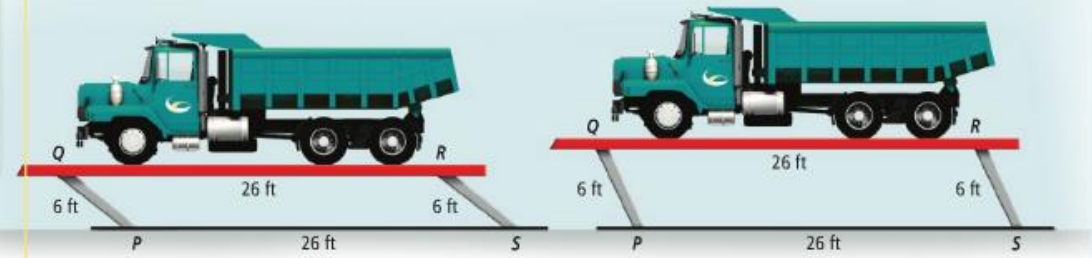


Problem 3 Identifying Parallelograms

Think

As the arms of the lift move, what changes and what stays the same?
The angles the arms form with the ground and the platform change, but the lengths of the arms and the platform stay the same.

Vehicle Lifts A truck sits on the platform of a vehicle lift. Two moving arms raise the platform until a mechanic can fit underneath. Why will the truck always remain parallel to the ground as it is lifted? Explain.



The angles of $PQRS$ change as platform $\overline{QR}$ rises, but its side lengths remain the same. Both pairs of opposite sides are congruent, so $PQRS$ is a parallelogram by Theorem 6-8. By the definition of a parallelogram, $\overline{PS} \parallel \overline{QR}$. Since the base of the lift $\overline{PS}$ lies along the ground, platform $\overline{QR}$, and therefore the truck, will always be parallel to the ground.

Got It? 3. **Reasoning** What is the maximum height that the vehicle lift can elevate the truck? Explain.

take note

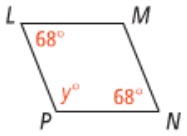
Concept Summary Proving That a Quadrilateral Is a Parallelogram

Method	Source	Diagram
Prove that both pairs of opposite sides are parallel.	Definition of parallelogram	
Prove that both pairs of opposite sides are congruent.	Theorem 6-8	
Prove that an angle is supplementary to both of its consecutive angles.	Theorem 6-9	75° 75° 105°
Prove that both pairs of opposite angles are congruent.	Theorem 6-10	
Prove that the diagonals bisect each other.	Theorem 6-11	
Prove that one pair of opposite sides is congruent and parallel.	Theorem 6-12	

Lesson Check

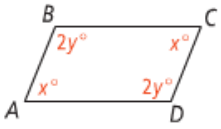
Do you know HOW?

1. For what value of y must $LMNP$ be a parallelogram?

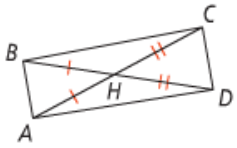


For Exercises 2 and 3, is the given information enough to prove that $ABCD$ is a parallelogram? Explain.

2.



3.



Do you UNDERSTAND?

4. **Vocabulary** Explain why you can now write a biconditional statement regarding opposite sides of a parallelogram.

5. **Compare and Contrast** How is Theorem 6-11 in this lesson different from Theorem 6-6 in the previous lesson? In what situations should you use each theorem? Explain.

6. **Error Analysis** Your friend says, "If a quadrilateral has a pair of opposite sides that are congruent and a pair of opposite sides that are parallel, then it is a parallelogram." What is your friend's error? Explain.

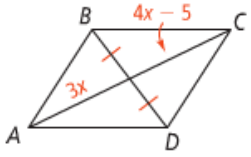
Practice and Problem-Solving Exercises

A Practice

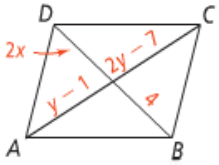
Algebra For what values of x and y must $ABCD$ be a parallelogram?

See Problems 1 and 2.

7.



8.



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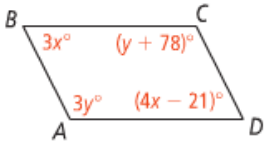
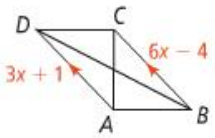
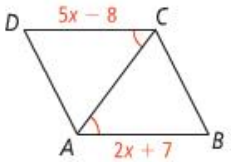


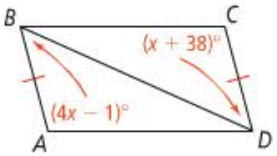
10.



11.

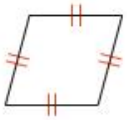


12.

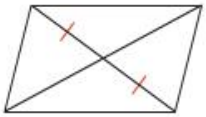


Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

13.

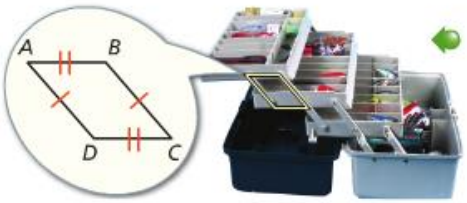


14.



15.

16. **Fishing** Quadrilaterals are formed on the side of this fishing tackle box by the adjustable shelves and connecting pieces. Explain why the shelves are always parallel to each other no matter what their position is.

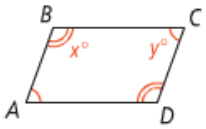


See Problem 3.

B Apply

17. Writing Combine each of Theorems 6-3, 6-4, 6-5, and 6-6 with its converse from this lesson into biconditional statements.

18. Developing Proof Complete this two-column proof of Theorem 6-10.



Given: $\angle A \cong \angle C$, $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.

Statements	Reasons
1) $x + y + x + y = 360$	1) The sum of the measures of the angles of a quadrilateral is 360.
2) $2(x + y) = 360$	2) a. ?
3) $x + y = 180$	3) b. ?
4) $\angle A$ and $\angle B$ are supplementary. $\angle A$ and $\angle D$ are supplementary.	4) Definition of supplementary
5) c. ? $\parallel$?, ? $\parallel$?	5) d. ?
6) $ABCD$ is a parallelogram.	6) e. ?

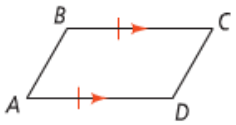
19. Think About a Plan Prove Theorem 6-12.

Proof

Given: $\overline{BC} \parallel \overline{DA}$, $\overline{BC} \cong \overline{DA}$

Prove: $ABCD$ is a parallelogram.

- How can drawing diagonals help you?
- How can you use triangles in this proof?

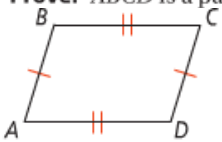


20. Prove Theorem 6-8.

Proof

Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$

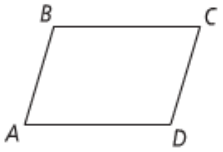
Prove: $ABCD$ is a parallelogram.



21. Prove Theorem 6-9.

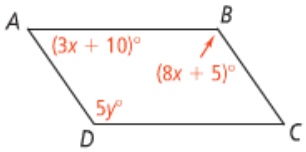
Proof

Given: $\angle A$ is supplementary to $\angle B$.
$\angle A$ is supplementary to $\angle D$.

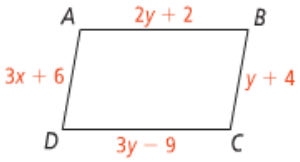


Algebra **For what values of the variables must $ABCD$ be a parallelogram?**

22.



23.



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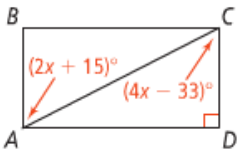
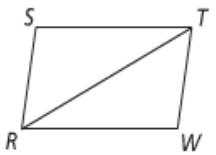


25. Proof

Given: $\triangle TRS \cong \triangle RTW$

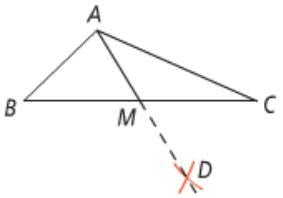
Prove: $RSTW$ is a parallelogram.



26. Open-Ended Sketch two noncongruent parallelograms $ABCD$ and $EFGH$ such that $\overline{AC} \cong \overline{EG}$ and $\overline{BD} \cong \overline{FH}$.

Challenge

27. Construction Proof In the figure at the right, point D is constructed by drawing two arcs. One has center C and radius AB. The other has center B and radius AC. Prove that $\overline{AM}$ is a median of $\triangle ABC$.



28. Probability If two opposite angles of a quadrilateral measure 120 and the measures of the other angles are multiples of 10, what is the probability that the quadrilateral is a parallelogram?

Standardized Test Prep

SAT/ACT

29. From which set of information can you conclude that $RSTW$ is a parallelogram?

(A) $\overline{RS} \parallel \overline{WT}, \overline{RS} \cong \overline{ST}$

(B) $\overline{RS} \parallel \overline{WT}, \overline{ST} \cong \overline{RW}$

(C) $\overline{RS} \cong \overline{ST}, \overline{RW} \cong \overline{WT}$

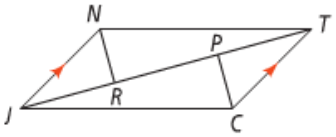
(D) $\overline{RZ} \cong \overline{TZ}, \overline{SZ} \cong \overline{WZ}$

Short Response

30. Write a proof using the diagram.

Given: $\triangle NRJ \cong \triangle CPT, \overline{JN} \parallel \overline{CT}$

Prove: $JNTC$ is a parallelogram.



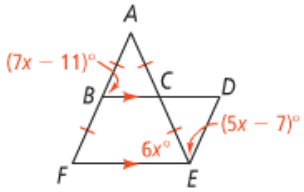
Extended Response

31. Use the figure at the right.

a. Write an equation and solve for x.

b. Is $\overline{AF} \parallel \overline{DE}$? Explain.

c. Is $BDEF$ a parallelogram? Explain.

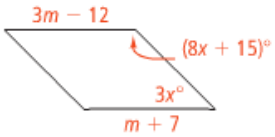


Mixed Review

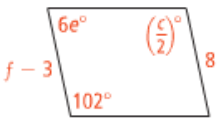
Algebra Find the value of each variable in each parallelogram. See Lesson 6-2.

32.

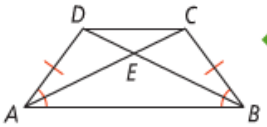
33.



34.



35. Explain how you can use overlapping congruent triangles to prove $\overline{AC} \cong \overline{BD}$. See Lessons 4-4 and 4-7.



Get Ready! **To prepare for Lesson 6-4, do Exercises 36–44.**

$PACE$ is a parallelogram and $m\angle PAC = 124$. Complete the following. See Lessons 5-2 and 6-2.

36. $AC =$ ■

37. $CE =$ ■

38. $PA =$ ■

39. $RE =$ ■

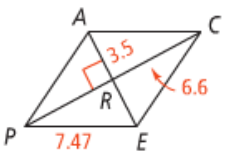
40. $CP =$ ■

41. $m\angle CEP =$ ■

42. $m\angle EPA =$ ■

43. $m\angle ECA =$ ■

44. $m\angle ACR =$ ■



6-4 Properties of Rhombuses, Rectangles, and Squares

Objectives To define and classify special types of parallelograms
To use properties of diagonals of rhombuses and rectangles

Notice that the starting and ending figures are both parallelograms.

SOLVE IT!

Getting Ready!

Fold a piece of notebook paper in half. Fold it in half again in the other direction. Draw a diagonal line from one vertex to the other. Cut through the folded paper along that line. Unfold the paper. What do you notice about the sides and about the diagonals of the figure you formed?

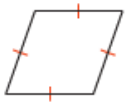
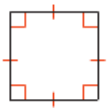
In the Solve It, you formed a special type of parallelogram with characteristics that you will study in this lesson.

Lesson Vocabulary
- rhombus
- rectangle
- square

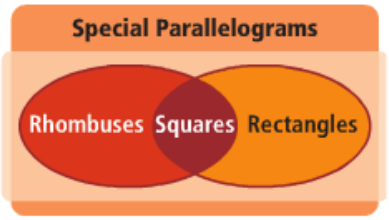
Essential Understanding The parallelograms in the Take Note box below have basic properties about their sides and angles that help identify them. The diagonals of these parallelograms also have certain properties.

take note

Key Concept Special Parallelograms

Definition	Diagram
A **rhombus** is a parallelogram with four congruent sides.	
A **rectangle** is a parallelogram with four right angles.	
A **square** is a parallelogram with four congruent sides and four right angles.	

The Venn diagram at the right shows the relationships among special parallelograms.



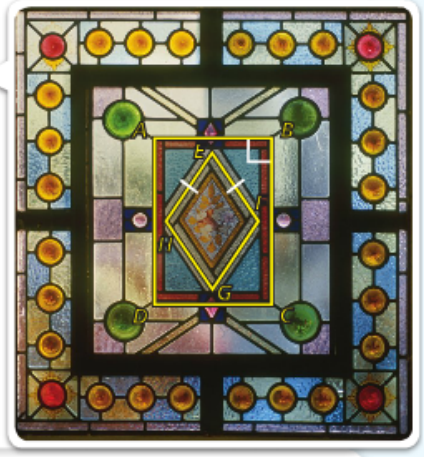
Problem 1 Classifying Special Parallelograms

Is $\square ABCD$ a rhombus, a rectangle, or a square? Explain.

$\square ABCD$ is a rectangle. Opposite angles of a parallelogram are congruent so $m\angle D$ is 90. By the Same-Side Interior Angles Theorem, $m\angle A = 90$ and $m\angle C = 90$. Since $\square ABCD$ has four right angles, it is a rectangle. You cannot conclude that $ABCD$ is a square because you do not know its side lengths.

Think

How do you decide whether *ABCD* is a rhombus, rectangle, or square?
Use the definitions of *rhombus*, *rectangle*, and *square* along with the markings on the figure.



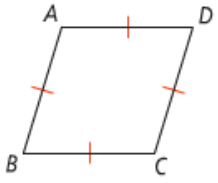
Got It? 1. Is $\square EFGH$ a rhombus, a rectangle, or a square? Explain.

take note

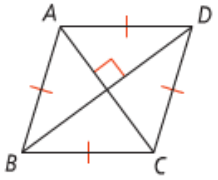
Theorem 6-13

Theorem
If a parallelogram is a rhombus, then its diagonals are perpendicular.

If . . .
ABCD is a rhombus



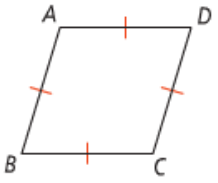
Then . . .
$\overline{AC} \perp \overline{BD}$



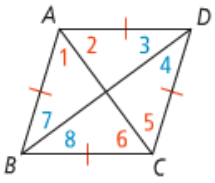
Theorem 6-14

Theorem
If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

If . . .
ABCD is a rhombus



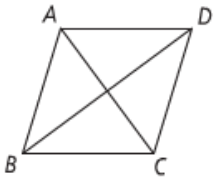
Then . . .



$\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
$\angle 5 \cong \angle 6$
$\angle 7 \cong \angle 8$

You will prove Theorem 6-14 in Exercise 45.

Proof **Proof of Theorem 6-13**



Given: $ABCD$ is a rhombus.

Prove: The diagonals of $ABCD$ are perpendicular.

Statements	Reasons
1) A and C are equidistant from B and D; B and D are equidistant from A and C.	1) All sides of a rhombus are $\cong$.
2) A and C are on the perpendicular bisector of $\overline{BD}$; B and D are on the perpendicular bisector of $\overline{AC}$.	2) Converse of the Perpendicular Bisector Theorem
3) $\overline{AC} \perp \overline{BD}$	3) Through two points, there is one unique line perpendicular to a given line.

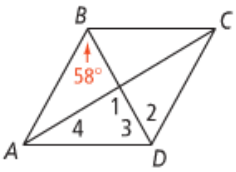
You can use Theorems 6-13 and 6-14 to find angle measures in a rhombus.

Problem 2 Finding Angle Measures

Think

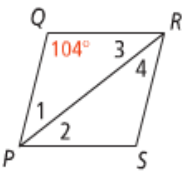
How are the numbered angles formed?
The angles are formed by diagonals. Use what you know about the diagonals of a rhombus to find the angle measures.

What are the measures of the numbered angles in rhombus $ABCD$?



$m\angle 1 = 90$	The diagonals of a rhombus are $\perp$.
$m\angle 2 = 58$	Alternate Interior Angles Theorem
$m\angle 3 = 58$	Each diagonal of a rhombus bisects a pair of opposite angles.
$m\angle 1 + m\angle 3 + m\angle 4 = 180$	Triangle Angle-Sum Theorem
$90 + 58 + m\angle 4 = 180$	Substitute.
$148 + m\angle 4 = 180$	Simplify.
$m\angle 4 = 32$	Subtract 148 from each side.

Got It? **2.** What are the measures of the numbered angles in rhombus $PQRS$?



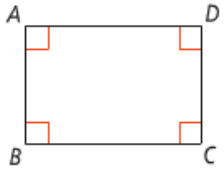
The diagonals of a rectangle also have a special property.

take note

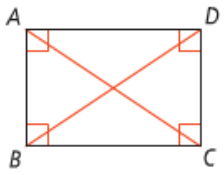
Theorem 6-15

Theorem
If a parallelogram is a rectangle, then its diagonals are congruent.

If . . .
$ABCD$ is a rectangle



Then . . .
$\overline{AC} \cong \overline{BD}$

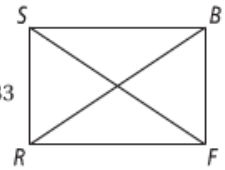


You will prove Theorem 6-15 in Exercise 41.

Problem 3 Finding Diagonal Length

Plan
How can you find the length of a diagonal?
Since *RSBF* is a rectangle and its diagonals are congruent, use the expressions to *write an equation*.

Multiple Choice In rectangle $RSBF$, $SF = 2x + 15$ and $RB = 5x - 12$. What is the length of a diagonal?



(A) 1 (B) 9 (C) 18 (D) 33

Think

You know that the diagonals of a rectangle are congruent, so their lengths are equal.

Write

$SF = RB$

Set the algebraic expressions for *SF* and *RB* equal to each other and find the value of *x*.

$$\begin{aligned} 2x + 15 &= 5x - 12 \\ 15 &= 3x - 12 \\ 27 &= 3x \\ 9 &= x \end{aligned}$$

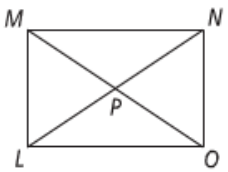
Substitute 9 for *x* in the expression for *RB*.

$$\begin{aligned} RB &= 5x - 12 \\ &= 5(9) - 12 \\ &= 33 \end{aligned}$$

The correct answer is D.

Got It? **3. a.** If $LN = 4x - 17$ and $MO = 2x + 13$, what are the lengths of the diagonals of rectangle $LMNO$?

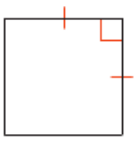
b. Reasoning What type of triangle is $\triangle PMN$? Explain.

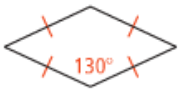


Lesson Check

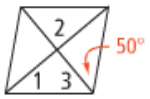
Do you know HOW?

Is each parallelogram a rhombus, rectangle, or square? Explain.

1. 

2. 

3. What are the measures of the numbered angles in the rhombus?

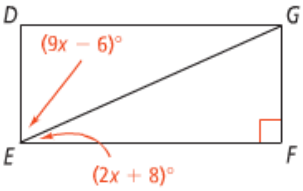


4. **Algebra** $JKLM$ is a rectangle. If $JL = 4x - 12$ and $MK = x$, what is the value of x? What is the length of each diagonal?

Do you UNDERSTAND?

5. **Vocabulary** Which special parallelograms are equiangular? Which special parallelograms are equilateral?

6. **Error Analysis** Your class needs to find the value of x for which $\square DEFG$ is a rectangle. A classmate's work is shown below. What is the error? Explain.



$2x + 8 = 9x - 6$
$14 = 7x$
$2 = x$

Practice and Problem-Solving Exercises

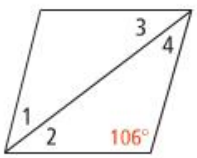
A Practice

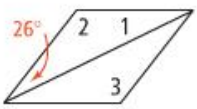
Decide whether the parallelogram is a rhombus, a rectangle, or a square. Explain. See Problem 1.

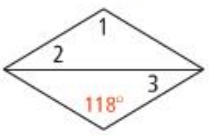
7.

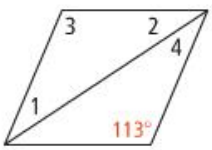
8.

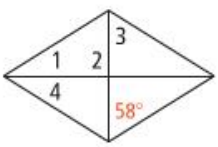
Find the measures of the numbered angles in each rhombus. See Problem 2.

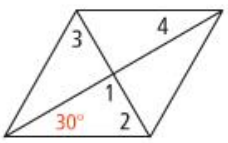
9. 

10. 

11. 

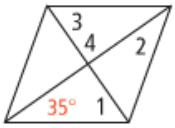
12. 

13. 

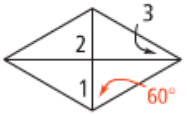
14. 

Find the measures of the numbered angles in each rhombus.

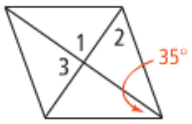
15.



16.



17.



Algebra *LMNP* **is a rectangle. Find the value of *x* and the length of each diagonal.** See Problem 3.

18. $LN = x$ and $MP = 2x - 4$

19. $LN = 5x - 8$ and $MP = 2x + 1$

20. $LN = 3x + 1$ and $MP = 8x - 4$

21. $LN = 9x - 14$ and $MP = 7x + 4$

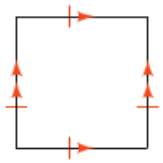
22. $LN = 7x - 2$ and $MP = 4x + 3$

23. $LN = 3x + 5$ and $MP = 9x - 10$

B Apply

Determine the most precise name for each quadrilateral.

24.



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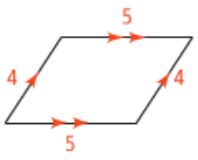
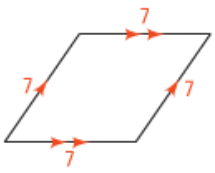
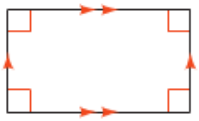


26.



27.



List the quadrilaterals that have the given property. Choose among *parallelogram*, *rhombus*, *rectangle*, and *square*.

28. All sides are $\cong$.

29. Opposite sides are $\cong$.

30. Opposite sides are $\parallel$.

31. Opposite $\measuredangle$ are $\cong$.

32. All $\measuredangle$ are right $\measuredangle$.

33. Consecutive $\measuredangle$ are supplementary.

34. Diagonals bisect each other.

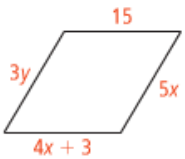
35. Diagonals are $\cong$.

36. Diagonals are $\perp$.

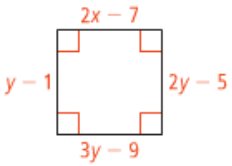
37. Each diagonal bisects opposite $\measuredangle$.

Algebra **Find the values of the variables. Then find the side lengths.**

38. rhombus



39. square



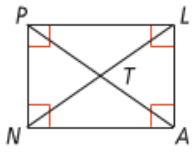
40. **Think About a Plan** Write a proof.

Proof

Given: Rectangle *PLAN*

Prove: $\triangle LTP \cong \triangle NTA$

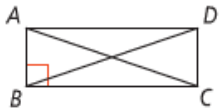
- What do you know about the diagonals of rectangles?
- Which triangle congruence postulate or theorem can you use?

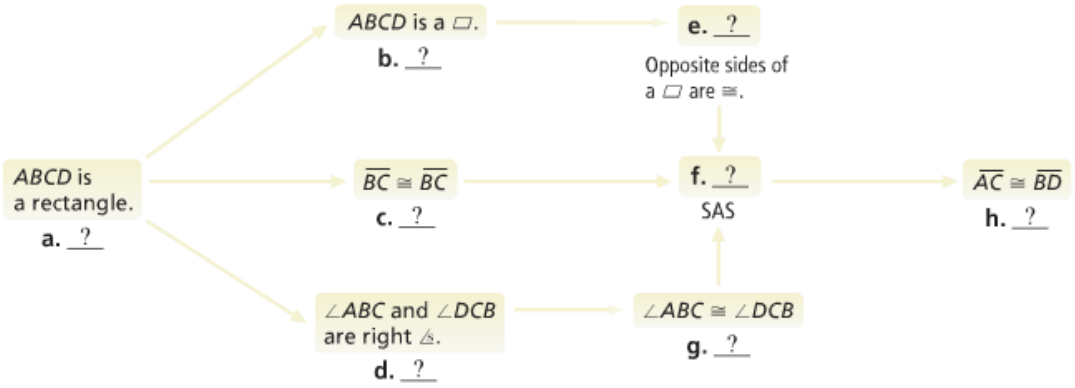


41. Developing Proof Complete the flow proof of Theorem 6-15.

Given: $ABCD$ is a rectangle.

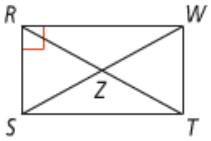
Prove: $\overline{AC} \cong \overline{BD}$



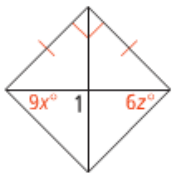


Algebra **Find the value(s) of the variable(s) for each parallelogram.**

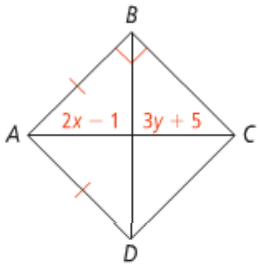
42. $RZ = 2x + 5$,
$SW = 5x - 20$



43. $m\angle 1 = 3y - 6$



44. $BD = 4x - y + 1$

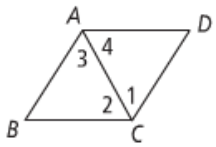


45. Prove Theorem 6-14.

Proof

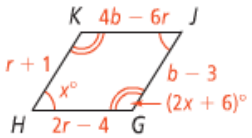
Given: $ABCD$ is a rhombus.

Prove: $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$.



46. Writing Summarize the properties of squares that follow from a square being (a) a parallelogram, (b) a rhombus, and (c) a rectangle.

47. Algebra Find the angle measures and the side lengths of the rhombus at the right.



48. Open-Ended On graph paper, draw a parallelogram that is neither a rectangle nor a rhombus.

Algebra ***ABCD* is a rectangle. Find the length of each diagonal.**

49. $AC = 2(x - 3)$ and $BD = x + 5$

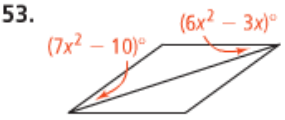
50. $AC = 2(5a + 1)$ and $BD = 2(a + 1)$

51. $AC = \frac{3y}{5}$ and $BD = 3y - 4$

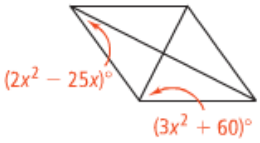
52. $AC = \frac{3c}{9}$ and $BD = 4 - c$

Challenge **Algebra** Find the value of x in the rhombus.

53.



54.



Standardized Test Prep

SAT/ACT

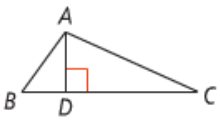
55. Which statement is true for some, but not all, rectangles?

(A) Opposite sides are parallel.
(B) It is a parallelogram.
(C) Adjacent sides are perpendicular.
(D) All sides are congruent.

56. A part of a design for a quilting pattern consists of a regular pentagon and five isosceles triangles, as shown. What is $m\angle 1$?

(F) 18
(G) 36
(H) 72
(I) 108

57. Which term best describes $\overline{AD}$ in $\triangle ABC$?



(A) altitude
(B) angle bisector
(C) median
(D) perpendicular bisector

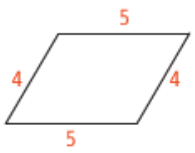
Short Response

58. Write the first step of an indirect proof that $\triangle PQR$ is not a right triangle.

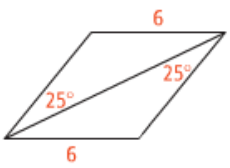
Mixed Review

Can you conclude that the quadrilateral is a parallelogram? Explain. See Lesson 6-3.

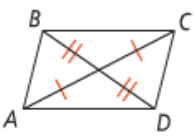
59.



60.

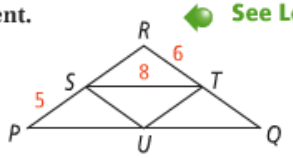


61.



In $\triangle PQR$, points S, T, and U are midpoints. Complete each statement. See Lesson 5-1.

62. $TQ = _?_$
63. $PQ = _?_$
64. $TU = _?_$
65. $\overline{SU} \parallel _?_$
66. $\overline{TU} \parallel _?_$
67. $\overline{PQ} \parallel _?_$



Get Ready! **To prepare for Lesson 6-5, do Exercises 68 and 69.**

68. Draw a rhombus that is not a square. See Lesson 6-4.

69. Draw a rectangle that is not a square.

6-5 Conditions for Rhombuses, Rectangles, and Squares

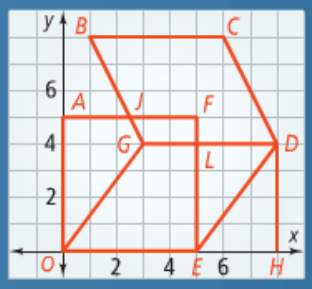
Objective To determine whether a parallelogram is a rhombus or rectangle

So far you have learned about four types of quadrilaterals.

SOLVE IT!

Getting Ready!

Which vertices form a square? A rhombus? A rectangle? Justify your answers.



Essential Understanding You can determine whether a parallelogram is a rhombus or a rectangle based on the properties of its diagonals.

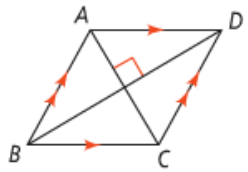
Dynamic Activity Special Parallelograms

take note

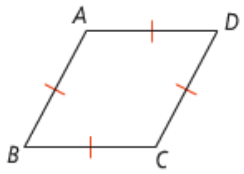
Theorem 6-16

Theorem
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

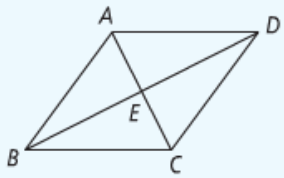
If . . .
$ABCD$ is a $\square$ and $\overline{AC} \perp \overline{BD}$



Then . . .
$ABCD$ is a rhombus



Proof **Proof of Theorem 6-16**



Given: $ABCD$ is a parallelogram, $\overline{AC} \perp \overline{BD}$

Prove: $ABCD$ is a rhombus.

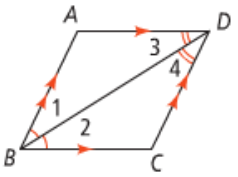
Since $ABCD$ is a parallelogram, $\overline{AC}$ and $\overline{BD}$ bisect each other, so $\overline{BE} \cong \overline{DE}$. Since $\overline{AC} \perp \overline{BD}$, $\angle AED$ and $\angle AEB$ are congruent right angles. By the Reflexive Property of Congruence, $\overline{AE} \cong \overline{AE}$. So $\triangle AEB \cong \triangle AED$ by SAS. Corresponding parts of congruent triangles are congruent, so $\overline{AB} \cong \overline{AD}$. Since opposite sides of a parallelogram are congruent, $\overline{AB} \cong \overline{DC} \cong \overline{BC} \cong \overline{AD}$. By definition, $ABCD$ is a rhombus.

take note

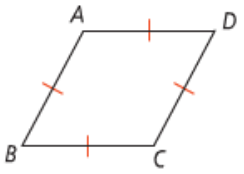
Theorem 6-17

Theorem
If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If . . .
$ABCD$ is a $\square$, $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$



Then . . .
$ABCD$ is a rhombus

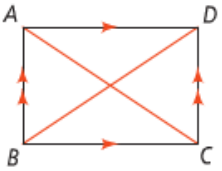


You will prove Theorem 6-17 in Exercise 23.

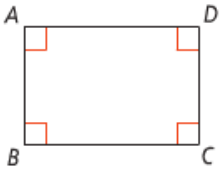
Theorem 6-18

Theorem
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

If . . .
$ABCD$ is a $\square$, and $\overline{AC} \cong \overline{BD}$



Then . . .
$ABCD$ is a rectangle



You will prove Theorem 6-18 in Exercise 24.

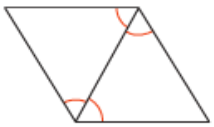
You can use Theorems 6-16, 6-17, and 6-18 to classify parallelograms. Notice that if a parallelogram is both a rectangle and a rhombus, then it is a square.

Problem 1 Identifying Special Parallelograms

Think
How can you determine whether a figure is a special parallelogram?
See if you can satisfy every condition of a definition or theorem about rhombuses, rectangles, or squares.

Can you conclude that the parallelogram is a rhombus, a rectangle, or a square? Explain.

A



Yes. A diagonal bisects two angles. By Theorem 6-17, this parallelogram is a rhombus.

B

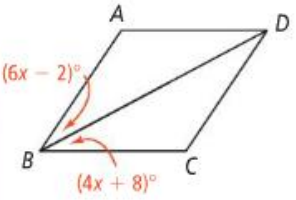
Yes. The diagonals are congruent, so by Theorem 6-18, this parallelogram is a rectangle. The diagonals are perpendicular, so by Theorem 6-16, it is a rhombus. Therefore, this parallelogram is a square.

Got It? **1. a.** A parallelogram has angle measures of 20, 160, 20, and 160. Can you conclude that it is a rhombus, a rectangle, or a square? Explain.

b. Reasoning Suppose the diagonals of a quadrilateral bisect each other. Can you conclude that it is a rhombus, a rectangle, or a square? Explain.

Problem 2 Using Properties of Special Parallelograms

Algebra For what value of x is $\square ABCD$ a rhombus?



Think

For $\square ABCD$ to be a rhombus, its diagonals must bisect a pair of opposite angles.

Write

$m\angle ABD = m\angle CBD$

Set the expressions for $m\angle ABD$ and $m\angle CBD$ equal to each other.

$6x - 2 = 4x + 8$

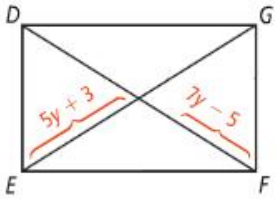
Solve for x.

$2x - 2 = 8$

$2x = 10$

$x = 5$

Got It? 2. For what value of y is $\square DEFG$ a rectangle?



Problem 3 Using Properties of Parallelograms

Community Service **Builders use properties of diagonals to "square up" rectangular shapes like building frames and playing-field boundaries. Suppose you are on the volunteer building team at the right. You are helping to lay out a rectangular patio for a youth center. How can you use properties of diagonals to locate the four corners?**

You can use two theorems.

- Theorem 6-11: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- Theorem 6-18: If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Think

Is there only one rectangle that can be formed by pulling the ropes taut?
No, you can change the shape of the rectangle. Have two of the people move closer together. Then the other two people move until the ropes are taut again.

Step 1 Cut two pieces of rope that will be the diagonals of the foundation rectangle. Cut them the same length because of Theorem 6-18.

Step 2 Join the two pieces of rope at their midpoints because of Theorem 6-11.

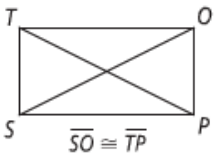
Step 3 Pull the ropes straight and taut. The ends of the ropes will be the corners of a rectangle.

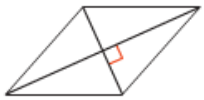
Got It? 3. Can you adapt this method slightly to stake off a square play area? Explain.

Lesson Check

Do you know HOW?

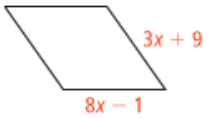
Can you conclude that the parallelogram is a rhombus, a rectangle, or a square? Explain.

1. 

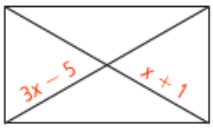
2. 

For what value of x is the figure the given special parallelogram?

3. rhombus



4. rectangle



Do you UNDERSTAND?

5. Name all of the special parallelograms that have each property.
   - **a.** Diagonals are perpendicular.
   - **b.** Diagonals are congruent.
   - **c.** Diagonals are angle bisectors.
   - **d.** Diagonals bisect each other.
   - **e.** Diagonals are perpendicular bisectors of each other.

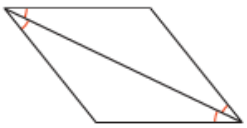
6. **Error Analysis** Your friend says, "A parallelogram with perpendicular diagonals is a rectangle." What is your friend's error? Explain.

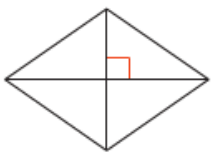
7. **Reasoning** When you draw a circle and two of its diameters and connect the endpoints of the diameters, what quadrilateral do you get? Explain.

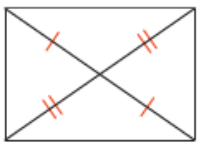
Practice and Problem-Solving Exercises

A Practice

Can you conclude that the parallelogram is a rhombus, a rectangle, or a square? Explain. See Problem 1.

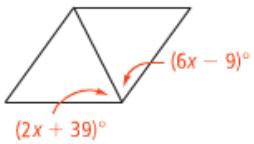
8. 

9. 

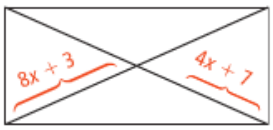
10. 

For what value of x is the figure the given special parallelogram? See Problem 2.

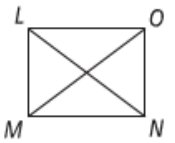
11. rhombus



12. rectangle



13. rectangle



$LN = 4x - 7$
$MO = 2x + 13$

14. **Carpentry** A carpenter is building a bookcase. How can the carpenter use a tape measure to check that the bookshelf is rectangular? Justify your answer and name any theorems used. See Problem 3.

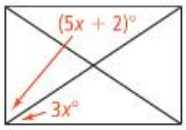
B Apply

15. Hardware You can use a simple device called a turnbuckle to "square up" structures that are parallelograms. For the gate pictured at the right, you tighten or loosen the turnbuckle on the diagonal cable so that the rectangular frame will keep the shape of a parallelogram when it sags. What are two ways you can make sure that the turnbuckle works? Explain.

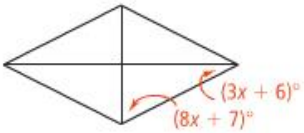
16. Reasoning Suppose the diagonals of a parallelogram are both perpendicular and congruent. What type of special quadrilateral is it? Explain your reasoning.

Algebra For what value of x is the figure the given special parallelogram?

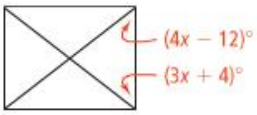
17. rectangle



18. rhombus



19. rectangle



Open-Ended Given two segments with lengths a and b $(a \neq b)$, what special parallelograms meet the given conditions? Show each sketch.

20. Both diagonals have length a.

21. The two diagonals have lengths a and b.

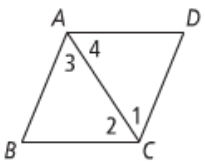
22. One diagonal has length a, and one side of the quadrilateral has length b.

23. Prove Theorem 6-17.

Proof

Given: $ABCD$ is a parallelogram.
$\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$.

Prove: $ABCD$ is a rhombus.

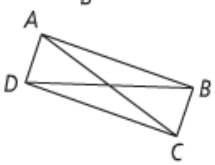


24. Prove Theorem 6-18.

Proof

Given: $\square ABCD$, $\overline{AC} \cong \overline{BD}$

Prove: $ABCD$ is a rectangle.



Think About a Plan Explain how to construct each figure given its diagonals.

- What do you know about the diagonals of each figure?
- How can you apply constructions to what you know about the diagonals?

25. parallelogram

26. rectangle

27. rhombus

C Challenge

Determine whether the quadrilateral can be a parallelogram. Explain.

28. The diagonals are congruent, but the quadrilateral has no right angles.

29. Each diagonal is 3 cm long and two opposite sides are 2 cm long.

30. Two opposite angles are right angles, but the quadrilateral is not a rectangle.

31. In Theorem 6-17, replace "a pair of opposite angles" with "one angle." Write a paragraph that proves this new statement to be true, or give a counterexample to prove it to be false. Proof

Standardized Test Prep

SAT/ACT

32. Each diagonal of a quadrilateral bisects a pair of opposite angles of the quadrilateral. What is the most precise name for the quadrilateral?

(A) parallelogram (B) rhombus (C) rectangle (D) not enough information

33. Given a triangle with side lengths 7 and 11, which value could NOT be the length of the third side of the triangle?

(F) 13 (G) 7 (H) 5 (I) 2

34. What is the sum of the measures of the exterior angles in a pentagon?

(A) 180 (B) 360 (C) 540 (D) 108

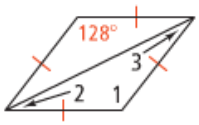
Short Response

35. The midpoint of $\overline{PQ}$ is $(-1, 4)$. One endpoint is $P(-7, 10)$. What are the coordinates of endpoint Q? Explain your work.

Mixed Review

Find the measures of the numbered angles in each rhombus. See Lesson 6-4.

36.



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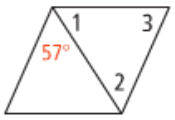

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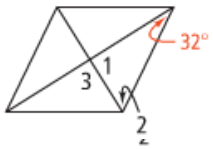


Write the two conditionals as a biconditional. See Lesson 2-3.

39. If a parallelogram is a rhombus, then its diagonals are perpendicular.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

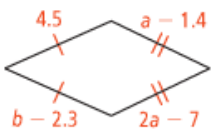
40. If a parallelogram is a rectangle, then its diagonals are congruent.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Get Ready! **To prepare for Lesson 6-6, do Exercises 41–43.**

Algebra **Find the values of the variables. Then find the lengths of the sides.** See Lesson 1-3.

41.



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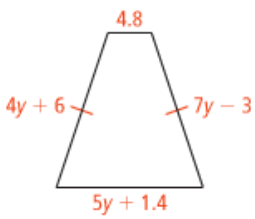
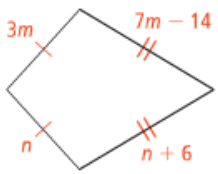


43.



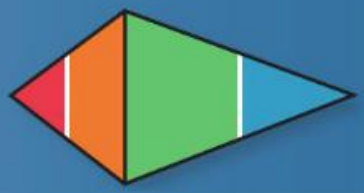
6-6 Trapezoids and Kites

Objective To verify and use properties of trapezoids and kites

Make a sketch and number the angles.

Getting Ready!

Two isosceles triangles form the figure at the right. Each white segment is a midsegment of a triangle. What can you determine about the angles in the orange region? In the green region? Explain.



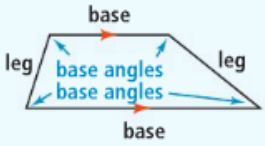
Dynamic Activity
Classifying Quadrilaterals

In the Solve It, the orange and green regions are trapezoids. The entire figure is a kite. In this lesson, you will learn about these special quadrilaterals that are not parallelograms.

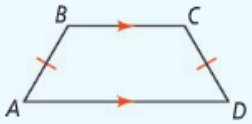
Essential Understanding The angles, sides, and diagonals of a trapezoid have certain properties.

Lesson Vocabulary
- trapezoid
- base
- leg
- base angle
- isosceles trapezoid
- midsegment of a trapezoid
- kite

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called **bases.** The nonparallel sides are called **legs.** The two angles that share a base of a trapezoid are called **base angles.** A trapezoid has two pairs of base angles.



An **isosceles trapezoid** is a trapezoid with legs that are congruent. *ABCD* at the right is an isosceles trapezoid. The angles of an isosceles trapezoid have some unique properties.

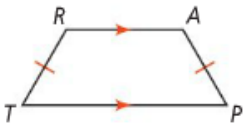


take note

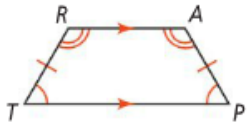
Theorem 6-19

Theorem
If a quadrilateral is an isosceles trapezoid, then each pair of base angles is congruent.

If . . .
TRAP is an isosceles trapezoid with bases $\overline{RA}$ and $\overline{TP}$



Then . . .
$\angle T \cong \angle P$, $\angle R \cong \angle A$



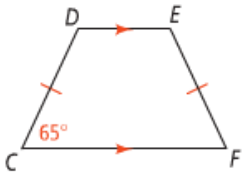
You will prove Theorem 6-19 in Exercise 45.

Problem 1 Finding Angle Measures in Trapezoids

Think

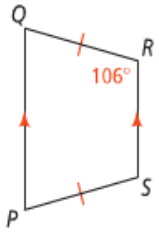
What do you know about the angles of an isosceles trapezoid?
You know that each pair of base angles is congruent. Because the bases of a trapezoid are parallel, you also know that two angles that share a leg are supplementary.

***CDEF* is an isosceles trapezoid and $m\angle C = 65$. What are $m\angle D$, $m\angle E$, and $m\angle F$?**



$m\angle C + m\angle D = 180$	Two angles that form same-side interior angles along one leg are supplementary.
$65 + m\angle D = 180$	Substitute.
$m\angle D = 115$	Subtract 65 from each side.

Since each pair of base angles of an isosceles trapezoid is congruent, $m\angle C = m\angle F = 65$ and $m\angle D = m\angle E = 115$.



Got It? **1. a.** In the diagram, *PQRS* is an isosceles trapezoid and $m\angle R = 106$. What are $m\angle P$, $m\angle Q$, and $m\angle S$?

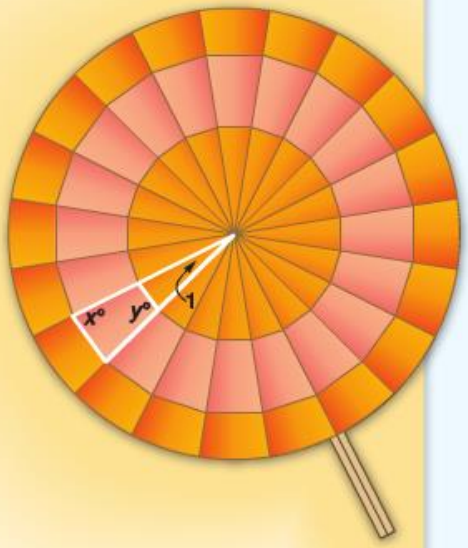
b. Reasoning In Problem 1, if *CDEF* were not an isosceles trapezoid, would $\angle C$ and $\angle D$ still be supplementary? Explain.

Problem 2 Finding Angle Measures in Isosceles Trapezoids

Think

What do you notice about the diagram?
Each trapezoid is part of an isosceles triangle with base angles that are the acute base angles of the trapezoid.

Paper Fans **The second ring of the paper fan shown at the right consists of 20 congruent isosceles trapezoids that appear to form circles. What are the measures of the base angles of these trapezoids?**



Step 1 Find the measure of each angle at the center of the fan. This is the measure of the vertex angle of an isosceles triangle.

$$m\angle 1 = \frac{360}{20} = 18$$

Step 2 Find the measure of each acute base angle of an isosceles triangle.

$18 + x + x = 180$	Triangle Angle-Sum Theorem
$18 + 2x = 180$	Combine like terms.
$2x = 162$	Subtract 18 from each side.
$x = 81$	Divide each side by 2.

Step 3 Find the measure of each obtuse base angle of the isosceles trapezoid.

$81 + y = 180$	Two angles that form same-side interior angles along one leg are supplementary.
$y = 99$	Subtract 81 from each side.

Each acute base angle measures 81. Each obtuse base angle measures 99.

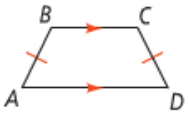
Got It? **2.** A fan like the one in Problem 2 has 15 angles meeting at the center. What are the measures of the base angles of the trapezoids in its second ring?

take note

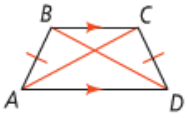
Theorem 6-20

Theorem
If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent.

If . . .
$ABCD$ is an isosceles trapezoid



Then . . .
$\overline{AC} \cong \overline{BD}$



You will prove Theorem 6-20 in Exercise 54.

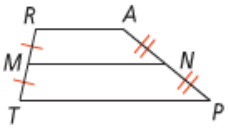
In Lesson 5-1, you learned about midsegments of triangles. Trapezoids also have midsegments. The **midsegment of a trapezoid** is the segment that joins the midpoints of its legs. The midsegment has two unique properties.

take note

Theorem 6-21 Trapezoid Midsegment Theorem

Theorem
If a quadrilateral is a trapezoid, then
(1) the midsegment is parallel to the bases, and
(2) the length of the midsegment is half the sum of the lengths of the bases.

If . . .
$TRAP$ is a trapezoid with midsegment $\overline{MN}$

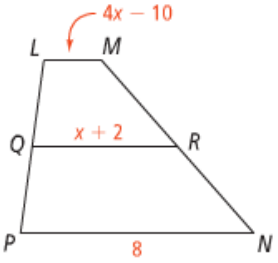


Then . . .
(1) $\overline{MN} \parallel \overline{TP}$, $\overline{MN} \parallel \overline{RA}$, and
(2) $MN = \frac{1}{2}\left(TP + RA\right)$

You will prove Theorem 6-21 in Lesson 6-9.

Problem 3 Using the Midsegment of a Trapezoid

Algebra $\overline{QR}$ **is the midsegment of trapezoid** $LMNP$.
What is x**?**



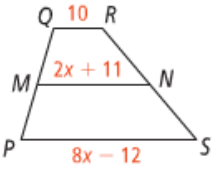
$QR = \frac{1}{2}(LM + PN)$	Trapezoid Midsegment Theorem
$x + 2 = \frac{1}{2}[(4x - 10) + 8]$	Substitute.
$x + 2 = \frac{1}{2}(4x - 2)$	Simplify.
$x + 2 = 2x - 1$	Distributive Property
$3 = x$	Subtract x and add 1 to each side.

Think

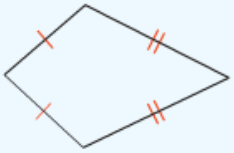
How can you check your answer?
Find LM and QR. Then see if QR equals half of the sum of the base lengths.

Got It? 3. a. **Algebra** $\overline{MN}$ is the midsegment of trapezoid $PQRS$. What is x? What is MN?

b. **Reasoning** How many midsegments can a triangle have? How many midsegments can a trapezoid have? Explain.



A **kite** is a quadrilateral with two pairs of consecutive sides congruent and no opposite sides congruent.



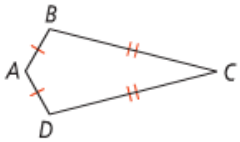
Essential Understanding The angles, sides, and diagonals of a kite have certain properties.

take note

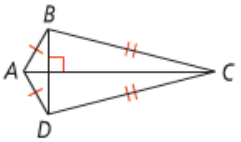
Theorem 6-22

Theorem
If a quadrilateral is a kite, then its diagonals are perpendicular.

If . . .
$ABCD$ is a kite



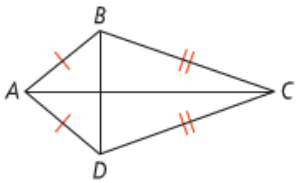
Then . . .
$\overline{AC} \perp \overline{BD}$



Proof **Proof of Theorem 6-22**

Given: Kite $ABCD$ with $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$

Prove: $\overline{AC} \perp \overline{BD}$



Statements	Reasons
1) Kite $ABCD$ with $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$	1) Given
2) A and C lie on the perpendicular bisector of $\overline{BD}$.	2) Converse of Perpendicular Bisector Theorem
3) $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.	3) Two points determine a line.
4) $\overline{AC} \perp \overline{BD}$	4) Definition of perpendicular bisector

Problem 4 Finding Angle Measures in Kites

Quadrilateral *DEFG* is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?

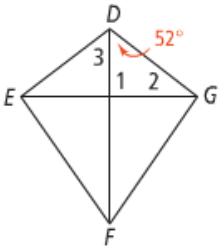
$m\angle 1 = 90$ Diagonals of a kite are $\perp$.

$90 + m\angle 2 + 52 = 180$ Triangle Angle-Sum Theorem

$142 + m\angle 2 = 180$ Simplify.

$m\angle 2 = 38$ Subtract 142 from each side.

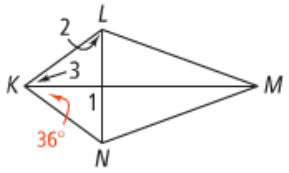
$\triangle DEF \cong \triangle DGF$ by SSS. Since corresponding parts of congruent triangles are congruent, $m\angle 3 = m\angle GDF = 52$.



Think

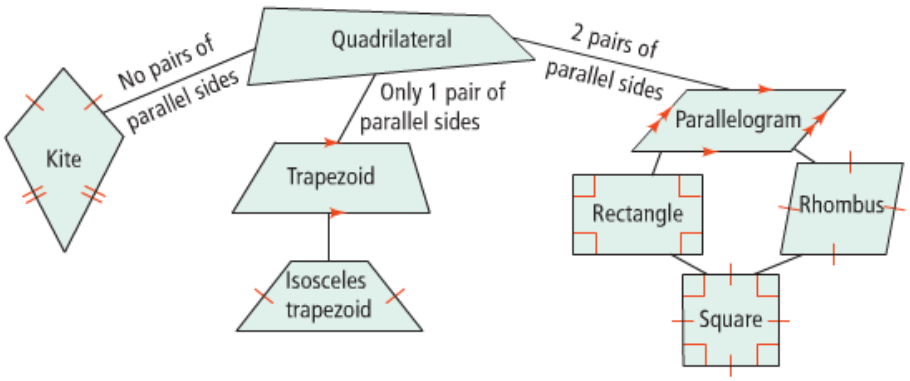
How are the triangles congruent by SSS? $\overline{DE} \cong \overline{DG}$ and $\overline{FE} \cong \overline{FG}$ because a kite has congruent consecutive sides. $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence.

Got It? **4.** Quadrilateral *KLMN* is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?



Take Note

Concept Summary Relationships Among Quadrilaterals

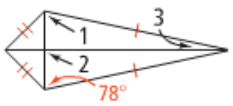


Lesson Check

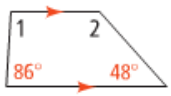
Do you know HOW?

What are the measures of the numbered angles?

1.



2.



3. What is the length of the midsegment of a trapezoid with bases of length 14 and 26?

Do you UNDERSTAND?

4. Vocabulary Is a kite a parallelogram? Explain.

5. Compare and Contrast How is a kite similar to a rhombus? How is it different? Explain.

6. Error Analysis Since a parallelogram has two pairs of parallel sides, it certainly has one pair of parallel sides. Therefore, a parallelogram must also be a trapezoid. What is the error in this reasoning? Explain.

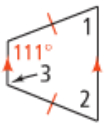
Practice and Problem-Solving Exercises

A Practice

Find the measures of the numbered angles in each isosceles trapezoid. See Problems 1 and 2.

7.

8.



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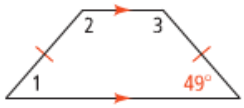

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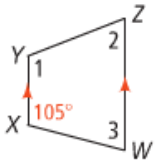

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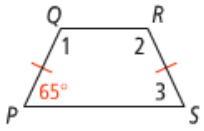

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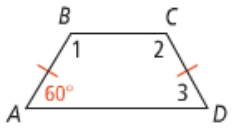
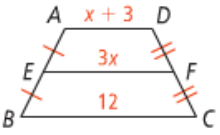


Find EF in each trapezoid. See Problem 3.

13.



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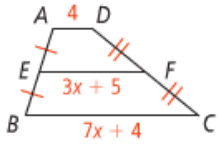

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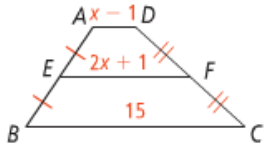
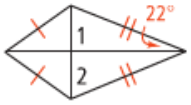


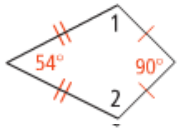
Find the measures of the numbered angles in each kite. See Problem 4.

16.



17.

18.



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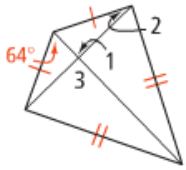

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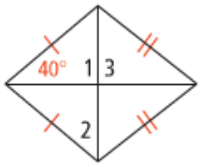

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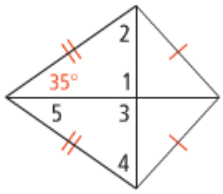

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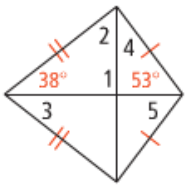

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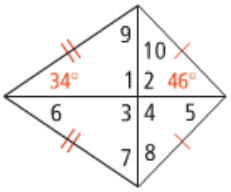

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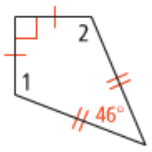


B Apply

25. Open-Ended Sketch two noncongruent kites such that the diagonals of one are congruent to the diagonals of the other.

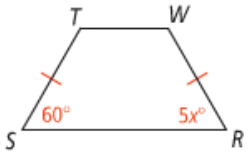
26. Think About a Plan The perimeter of a kite is 66 cm. The length of one of its sides is 3 cm less than twice the length of another. Find the length of each side of the kite.

- Can you draw a diagram?
- How can you write algebraic expressions for the lengths of the sides?

27. Reasoning If $KLMN$ is an isosceles trapezoid, is it possible for $\overline{KM}$ to bisect $\angle LMN$ and $\angle LKN$? Explain.

Algebra Find the value of the variable in each isosceles trapezoid.

28.



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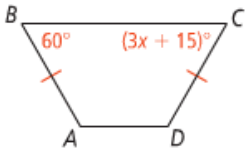

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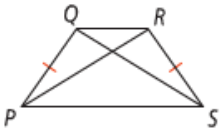
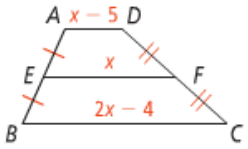


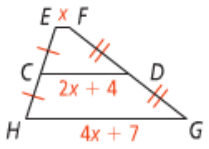
$QS = x + 5$
$RP = 3x + 3$

Algebra Find the lengths of the segments with variable expressions.

31.



32.



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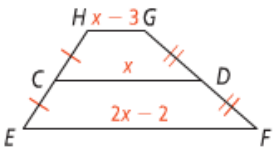
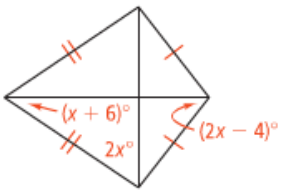


Algebra Find the value(s) of the variable(s) in each kite.

34.



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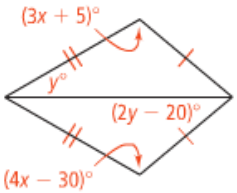
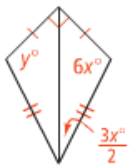
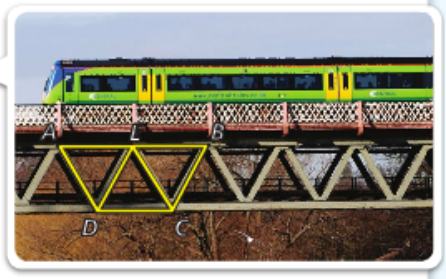


36.



Bridge Design The beams of the bridge at the right form quadrilateral $ABCD$. $\triangle AED \cong \triangle CDE \cong \triangle BEC$ and $m\angle DCB = 120$.



37. Classify the quadrilateral. Explain your reasoning.

38. Find the measures of the other interior angles of the quadrilateral.

Reasoning Can two angles of a kite be as follows? Explain.

39. opposite and acute

40. consecutive and obtuse

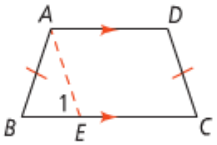
41. opposite and supplementary

42. consecutive and supplementary

43. opposite and complementary

44. consecutive and complementary

45. Developing Proof The plan suggests a proof of Theorem 6-19. Write a proof that follows the plan.



Given: Isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{DC}$

Prove: $\angle B \cong \angle C$ and $\angle BAD \cong \angle D$

Plan: Begin by drawing $\overline{AE} \parallel \overline{DC}$ to form parallelogram $AECD$ so that $\overline{AE} \cong \overline{DC} \cong \overline{AB}$. $\angle B \cong \angle C$ because $\angle B \cong \angle 1$ and $\angle 1 \cong \angle C$. Also, $\angle BAD \cong \angle D$ because they are supplements of the congruent angles, $\angle B$ and $\angle C$.

46. Proof Prove the converse of Theorem 6-19: If a trapezoid has a pair of congruent base angles, then the trapezoid is isosceles.

Name each type of special quadrilateral that can meet the given condition. Make sketches to support your answers.

47. exactly one pair of congruent sides

48. two pairs of parallel sides

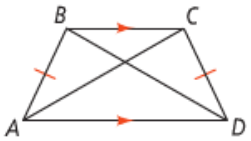
49. four right angles

50. adjacent sides that are congruent

51. perpendicular diagonals

52. congruent diagonals

53. Proof Prove Theorem 6-20.



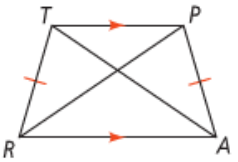
Given: Isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{DC}$

Prove: $\overline{AC} \cong \overline{DB}$

54. Proof Prove the converse of Theorem 6-20: If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.

55. Proof **Given:** Isosceles trapezoid $TRAP$ with $\overline{TR} \cong \overline{PA}$

Prove: $\angle RTA \cong \angle APR$



56. Proof Prove that the angles formed by the noncongruent sides of a kite are congruent. (*Hint:* Draw a diagonal of the kite.)

Determine whether each statement is *true* or *false*. Justify your response.

57. All squares are rectangles.

58. A trapezoid is a parallelogram.

59. A rhombus can be a kite.

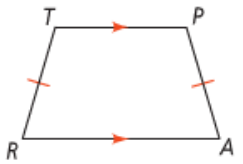
60. Some parallelograms are squares.

61. Every quadrilateral is a parallelogram.

62. All rhombuses are squares.

Challenge

63. Proof **Given:** Isosceles trapezoid $TRAP$ with $\overline{TR} \cong \overline{PA}$; $\overline{BI}$ is the perpendicular bisector of $\overline{RA}$, intersecting $\overline{RA}$ at B and $\overline{TP}$ at I.



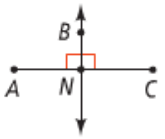
Prove: $\overline{BI}$ is the perpendicular bisector of $\overline{TP}$.

For a trapezoid, consider the segment joining the midpoints of the two given segments. How are its length and the lengths of the two parallel sides of the trapezoid related? Justify your answer.

64. the two nonparallel sides

65. the diagonals

66. $\overleftrightarrow{BN}$ is the perpendicular bisector of $\overline{AC}$ at N. Describe the set of points, D, for which $ABCD$ is a kite.



Standardized Test Prep

SAT/ACT

67. Which statement is never true?

(A) Square $ABCD$ is a rhombus.

(B) Trapezoid $GHJK$ is a parallelogram.

(C) Parallelogram $PQRS$ is a square.

(D) Square $WXYZ$ is a parallelogram.

68. A quadrilateral has four congruent sides. Which name best describes the figure?

(F) trapezoid (G) parallelogram (H) rhombus (I) kite

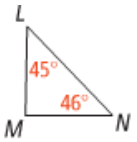
69. How would you classify triangle LMN?

(A) acute isosceles

(B) right isosceles

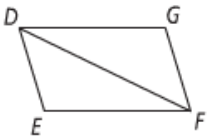
(C) obtuse scalene

(D) acute scalene



Extended Response

70. Given $\overline{DE}$ is congruent to $\overline{FG}$ and $\overline{EF}$ is congruent to $\overline{GD}$, prove $\angle E \cong \angle G$.

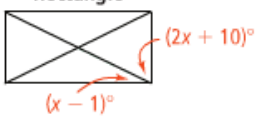


Mixed Review

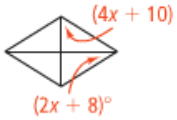
Find the value of x for which the figure is the given special parallelogram. See Lesson 6-5.

71. Rhombus

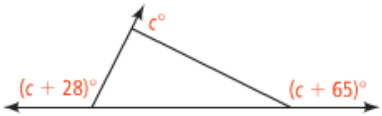
72. Rectangle



73. Rhombus



74. Algebra Find the value of c. See Lesson 6-1.



Get Ready! To prepare for Lesson 6-7, do Exercises 75 and 76.

75. Given $A(6, -2)$ and $B(-4, 8)$, find the midpoint and the length of $\overline{AB}$. See Lesson 1-7.

76. Find the slope of the line containing the points $C(-1, 5)$ and $D(7, 3)$. See Lesson 3-7.

6 Mid-Chapter Quiz

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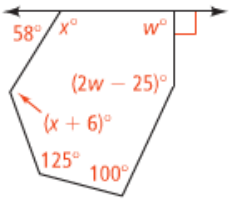
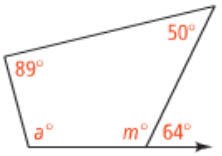
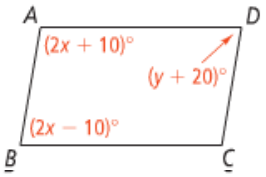
Do you know HOW?

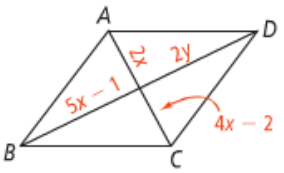
Find the value of each variable.

1. 

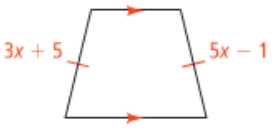
2. 

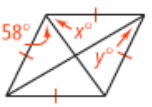
Algebra Find the values of the variables for which $ABCD$ is a parallelogram.

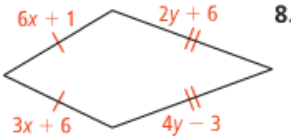
3. 

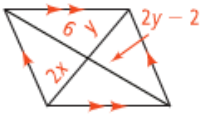
4. 

Algebra Classify the quadrilateral. Then find the value(s) of the variable(s).

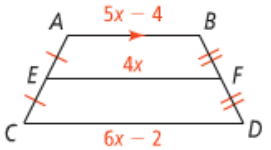
5. 

6. 

7. 

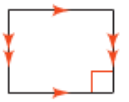
8. 

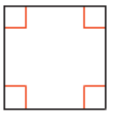
9. Find AB, CD, and EF.

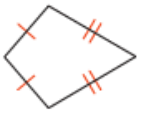


Classify the quadrilateral as precisely as possible. Explain your reasoning.

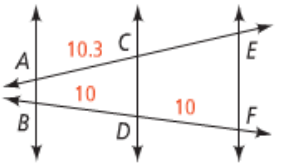
10.

11. 

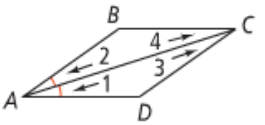
12. 

13. 

14. In the figure at the right, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$. Find AE.



15. **Given:** $\square ABCD$, $\overline{AC}$ bisects $\angle DAB$
Prove: $\overline{AC}$ bisects $\angle DCB$.



Do you UNDERSTAND?

Decide whether the statement is *true* or *false*. If true, explain why. If false, show a counterexample.

16. A quadrilateral with congruent diagonals is either an isosceles trapezoid or a rectangle.

17. A quadrilateral with congruent and perpendicular diagonals must be a kite.

18. Each diagonal of a kite bisects two angles of the kite.

19. **Reasoning** Can you fit all of the interior angles of a quadrilateral around a point without overlap? What about the interior angles of a pentagon? Explain.

20. **Writing** Explain two ways to show that a parallelogram is a rhombus.

21. Draw a diagram showing the relationships among the special quadrilaterals that you have learned.

Algebra Review

Use With Lesson 6-7

Simplifying Radicals

A radical expression is in simplest form when all of the following are true.

- The radicand has no perfect square factors other than 1.
- The radicand does not contain a fraction.
- A denominator does not contain a radical expression.

radical symbol ↓ $\sqrt{a}$ ← radicand

Example 1

Simplify the expressions $\sqrt{2} \cdot \sqrt{8}$ and $\sqrt{294} \div \sqrt{3}$.

$\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$ Write both numbers under one radical.

$= \sqrt{16}$ Simplify the expression under the radical.

$= 4$ Factor out perfect squares and simplify.

$\sqrt{294} \div \sqrt{3} = \sqrt{\frac{294}{3}}$

$= \sqrt{98}$

$= \sqrt{49 \cdot 2}$

$= 7\sqrt{2}$

Example 2

Write $\sqrt{\frac{4}{3}}$ in simplest form.

$\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}}$ Rewrite the single radical as a quotient.

$= \frac{2}{\sqrt{3}}$ Simplify the numerator.

$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$ (a form of 1) to remove the radical from the denominator. This is called *rationalizing the denominator*.

$= \frac{2\sqrt{3}}{3}$

Exercises

Simplify each expression.

1. $\sqrt{5} \cdot \sqrt{10}$
2. $\sqrt{243}$
3. $\sqrt{128} \div \sqrt{2}$
4. $\sqrt{\frac{125}{4}}$
5. $\sqrt{6} \cdot \sqrt{8}$
6. $\frac{\sqrt{36}}{\sqrt{3}}$
7. $\frac{\sqrt{144}}{\sqrt{2}}$
8. $\sqrt{3} \cdot \sqrt{12}$
9. $\sqrt{72} \div \sqrt{2}$
10. $\sqrt{169}$
11. $28 \div \sqrt{8}$
12. $\sqrt{300} \div \sqrt{5}$
13. $\sqrt{12} \cdot \sqrt{2}$
14. $\frac{\sqrt{6} \cdot \sqrt{3}}{\sqrt{9}}$
15. $\frac{\sqrt{3} \cdot \sqrt{15}}{\sqrt{2}}$

6-7 Polygons in the Coordinate Plane

Objective To classify polygons in the coordinate plane

Apply what you learned "B-4" about classifying polygons.

Getting Ready!

You and a friend are playing a board game. Players place rubber bands on their own square grid to form different shapes. The object of the game is to guess the vertices of your opponent's shape. How would you place pieces on the grid shown to complete a right isosceles triangle? Sketch the triangle and justify the placement of each piece.

In the Solve It, you formed a polygon on a grid. In this lesson, you will classify polygons in the coordinate plane.

Essential Understanding You can classify figures in the coordinate plane using the formulas for slope, distance, and midpoint.

The chart below reviews these formulas and tells when to use them.

take note

Key Concept Formulas and the Coordinate Plane

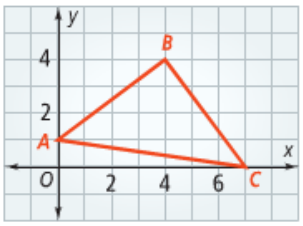
Formula	When to Use It
Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	To determine whether • sides are congruent • diagonals are congruent
Midpoint Formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	To determine • the coordinates of the midpoint of a side • whether diagonals bisect each other
Slope Formula $m = \frac{y_2 - y_1}{x_2 - x_1}$	To determine whether • opposite sides are parallel • diagonals are perpendicular • sides are perpendicular

Problem 1 Classifying a Triangle

Think

What formulas do you use?
The terms *scalene*, *isosceles*, and *equilateral* have to do with the side lengths of a triangle. Use the Distance Formula.

Is $\triangle ABC$ *scalene, isosceles,* or *equilateral*?



The vertices of the triangle are $A(0, 1)$, $B(4, 4)$, and $C(7, 0)$.

Find the lengths of the sides using the Distance Formula.

$$AB = \sqrt{(4 - 0)^2 + (4 - 1)^2}$$
$$= \sqrt{16 + 9} \quad \text{Simplify within parentheses. Then simplify the powers.}$$
$$= \sqrt{25} \quad \text{Simplify the radicand.}$$
$$= 5 \quad \text{Simplify.}$$

$$BC = \sqrt{(7 - 4)^2 + (0 - 4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

$$CA = \sqrt{(0 - 7)^2 + (1 - 0)^2}$$
$$= \sqrt{49 + 1}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

Since $AB = BC = 5$, $\triangle ABC$ is isosceles.

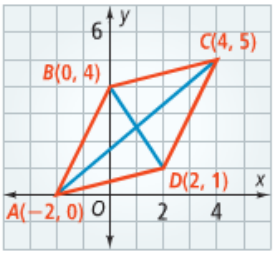
Got It? **1.** $\triangle DEF$ has vertices $D(0, 0)$, $E(1, 4)$, and $F(5, 2)$. Is $\triangle DEF$ *scalene, isosceles,* or *equilateral*?

Problem 2 Classifying a Parallelogram

Plan

How can you determine whether *ABCD* is a rhombus?
You can find the length of each side using the Distance Formula, or you can determine whether the diagonals are perpendicular. The second way requires fewer steps.

Is $\square ABCD$ a rhombus? Explain.



Step 1 Use the Slope Formula to find the slopes of the diagonals.

$$\text{slope of } \overline{AC} = \frac{5 - 0}{4 - (-2)} = \frac{5}{6}$$

$$\text{slope of } \overline{BD} = \frac{1 - 4}{2 - 0} = -\frac{3}{2}$$

Step 2 Find the product of the slopes.

$$\frac{5}{6} \cdot \left(-\frac{3}{2}\right) = -\frac{15}{12}$$

Since the product of the slopes is not -1, the diagonals are not perpendicular. So $ABCD$ is not a rhombus.

Got It? **2.** $\square MNPQ$ has vertices $M(0, 1)$, $N(-1, 4)$, $P(2, 5)$, and $Q(3, 2)$.

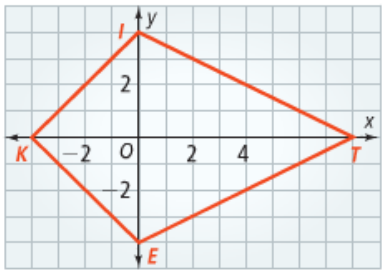
a. Is $\square MNPQ$ a rectangle? Explain.

b. Is $\square MNPQ$ a square? Explain.

c. Reasoning Is the triangle in Problem 1 a right triangle? Explain.

Problem 3 Classifying a Quadrilateral

A kite is shown at the right. What is the most precise classification of the quadrilateral formed by connecting the midpoints of the sides of the kite?



Know

$K(-4, 0)$, $I(0, 4)$, $T(8, 0)$, and $E(0, -4)$

Need

The midpoints of the sides of the kite

Plan

Use the Midpoint Formula to find the coordinates of the vertices of the inner quadrilateral. *Draw a diagram* to see it. Then classify the figure.

Step 1 Find the midpoint of each side of the kite.

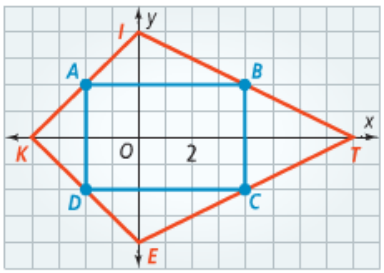
A = midpoint of $\overline{KI} = \left(\frac{-4 + 0}{2}, \frac{0 + 4}{2}\right) = (-2, 2)$

B = midpoint of $\overline{IT} = \left(\frac{0 + 8}{2}, \frac{4 + 0}{2}\right) = (4, 2)$

C = midpoint of $\overline{TE} = \left(\frac{8 + 0}{2}, \frac{0 + (-4)}{2}\right) = (4, -2)$

D = midpoint of $\overline{EK} = \left(\frac{0 + (-4)}{2}, \frac{-4 + 0}{2}\right) = (-2, -2)$

Step 2 Draw a diagram of $ABCD$.



Step 3 Classify $ABCD$.

$AB = |4 - (-2)| = 6$ Use the definition of distance on a number line. $BC = |-2 - 2| = 4$

$CD = |-2 - 4| = 6$ $DA = |2 - (-2)| = 4$

Since opposite sides are congruent, $ABCD$ is a parallelogram.

Since $\overline{AB}$ and $\overline{CD}$ are both horizontal, and $\overline{BC}$ and $\overline{DA}$ are both vertical, the segments form right angles. So, $ABCD$ is a rectangle.

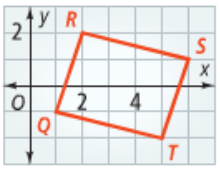
Got It? **3.** An isosceles trapezoid has vertices $A(0, 0)$, $B(2, 4)$, $C(6, 4)$, and $D(8, 0)$. What special quadrilateral is formed by connecting the midpoints of the sides of $ABCD$?

Lesson Check

Do you know HOW?

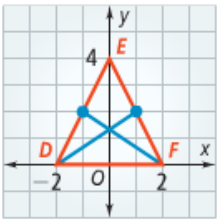
1. $\triangle TRI$ has vertices $T(-3, 4)$, $R(3, 4)$, and $I(0, 0)$. Is $\triangle TRI$ *scalene, isosceles,* or *equilateral*?

2. Is $QRST$ below a rectangle? Explain.



Do you UNDERSTAND?

3. **Writing** Describe how you would determine whether the lengths of the medians from base angles D and F are congruent.



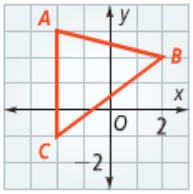
4. **Error Analysis** A student says that the quadrilateral with vertices $D(1, 2)$, $E(0, 7)$, $F(5, 6)$, and $G(7, 0)$ is a rhombus because its diagonals are perpendicular. What is the student's error?

Practice and Problem-Solving Exercises

A Practice

Determine whether $\triangle ABC$ is *scalene, isosceles,* or *equilateral*. Explain. See Problem 1.

5.



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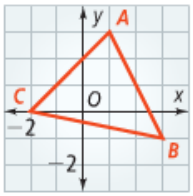

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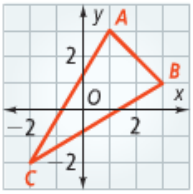


Determine whether the parallelogram is a *rhombus, rectangle, square,* or *none*. Explain. See Problem 2.

8. $P(-1, 2)$, $O(0, 0)$, $S(4, 0)$, $T(3, 2)$

9. $L(1, 2)$, $M(3, 3)$, $N(5, 2)$, $P(3, 1)$

10. $R(-2, -3)$, $S(4, 0)$, $T(3, 2)$, $V(-3, -1)$

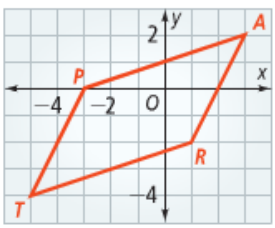
11. $G(0, 0)$, $H(6, 0)$, $I(9, 1)$, $J(3, 1)$

12. $W(-3, 0)$, $I(0, 3)$, $N(3, 0)$, $D(0, -3)$

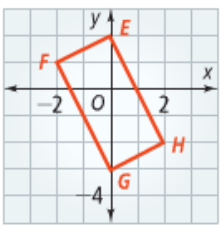
13. $S(1, 3)$, $P(4, 4)$, $A(3, 1)$, $T(0, 0)$

What is the most precise classification of the quadrilateral formed by connecting in order the midpoints of each figure below? See Problem 3.

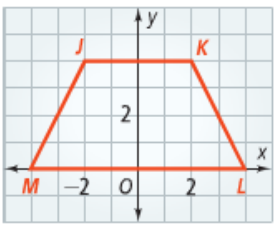
14. parallelogram *PART*



15. rectangle *EFGH*



16. isosceles trapezoid *JKLM*



B Apply

Graph and label each triangle with the given vertices. Determine whether each triangle is *scalene*, *isosceles*, or *equilateral*. Then tell whether each triangle is a right triangle.

17. $T(1, 1), R(3, 8), I(6, 4)$

18. $J(-5, 0), K(5, 8), L(4, -1)$

19. $A(3, 2), B(-10, 4), C(-5, -8)$

20. $H(1, -2), B(-1, 4), F(5, 6)$

Graph and label each quadrilateral with the given vertices. Then determine the most precise name for each quadrilateral.

21. $P(-5, 0), Q(-3, 2), R(3, 2), S(5, 0)$

22. $S(0, 0), T(4, 0), U(3, 2), V(-1, 2)$

23. $F(0, 0), G(5, 5), H(8, 4), I(7, 1)$

24. $M(-14, 4), N(1, 6), P(3, -9), Q(-12, -11)$

25. $A(3, 5), B(7, 6), C(6, 2), D(2, 1)$

26. $N(-6, 4), P(-3, 1), Q(0, 2), R(-3, 5)$

27. $J(2, 1), K(5, 4), L(8, 1), M(2, -3)$

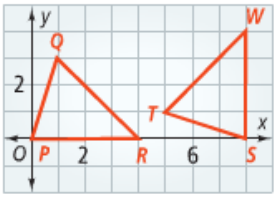
28. $H(-2, -3), I(4, 0), J(3, 2), K(-3, -1)$

29. $W(-1, 1), X(0, 2), Y(1, 1), Z(0, -2)$

30. $D(-3, 1), E(-7, -3), F(6, -3), G(2, 1)$

31. Think About a Plan Are the triangles at the right congruent? How do you know?

- Which triangle congruence theorem can you use?
- Which formula should you use?



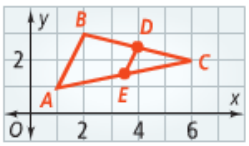
32. Reasoning A quadrilateral has opposite sides with equal slopes and consecutive sides with slopes that are negative reciprocals. What is the most precise classification of the quadrilateral? Explain.

Determine the most precise name for each quadrilateral. Then find its area.

33. $A(0, 2), B(4, 2), C(-3, -4), D(-7, -4)$

34. $J(1, -3), K(3, 1), L(7, -1), M(5, -5)$

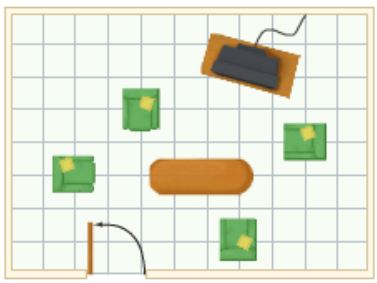
35. $\overline{DE}$ is a midsegment of $\triangle ABC$ at the right. Show that the Triangle Midsegment Theorem holds true for $\triangle ABC$.



36. a. Writing Describe two ways you can show whether a quadrilateral in the coordinate plane is a square.

b. Reasoning Which method is more efficient? Explain.

37. Interior Design Interior designers often use grids to plan the placement of furniture in a room. The design at the right shows four chairs around a coffee table. The designer plans for cutouts of chairs on lattice points. She wants the chairs oriented at the vertices of a parallelogram. Does she need to fix her plan? If so, describe the change(s) she should make.

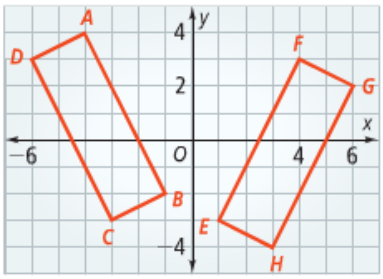


38. Use the diagram at the right.

a. What is the most precise classification of $ABCD$?

b. What is the most precise classification of $EFGH$?

c. Are $ABCD$ and $EFGH$ congruent? Explain.



Challenge

39. Coordinate Geometry The diagonals of quadrilateral $EFGH$ intersect at $D(-1, 4)$. $EFGH$ has vertices at $E(2, 7)$ and $F(-3, 5)$. What must be the coordinates of G and H to ensure that $EFGH$ is a parallelogram?

The endpoints of $\overline{AB}$ are $A(-3, 5)$ and $B(9, 15)$. Find the coordinates of the points that divide $\overline{AB}$ into the given number of congruent segments.

40. 4 **41.** 6 **42.** 10 **43.** 50 **44.** n

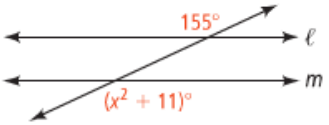
Standardized Test Prep

SAT/ACT

45. $K(-3, 0)$, $I(0, 2)$, and $T(3, 0)$ are three vertices of a kite. Which point could be the fourth vertex?

Ⓐ $E(0, 5)$ Ⓑ $E(0, 0)$ Ⓒ $E(0, -2)$ Ⓓ $E(0, -10)$

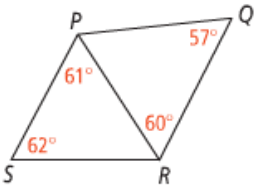
46. In the diagram, lines ℓ and m are parallel. What is the value of x?



Ⓕ 5 Ⓗ 13

Ⓖ 12 Ⓘ 25

47. In the diagram, which segment is shortest?



Ⓐ $\overline{PS}$ Ⓒ $\overline{PQ}$

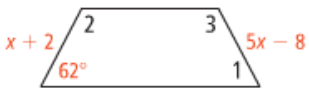
Ⓑ $\overline{PR}$ Ⓓ $\overline{QR}$

Short Response

48. $A(-3, 1)$, $B(-1, -2)$, and $C(2, 1)$ are three vertices of quadrilateral $ABCD$. Could $ABCD$ be a rectangle? Explain.

Mixed Review

49. Algebra Find the measure of each angle and the value of x in the isosceles trapezoid.



See Lesson 6-6.

Find the circumcenter of $\triangle ABC$.

See Lesson 5-3.

50. $A(1, 1)$, $B(5, 3)$, $C(5, 1)$ **51.** $A(-5, 0)$, $B(-1, -8)$, $C(-1, 0)$

Get Ready! **To prepare for Lesson 6-8, do Exercises 52–54.**

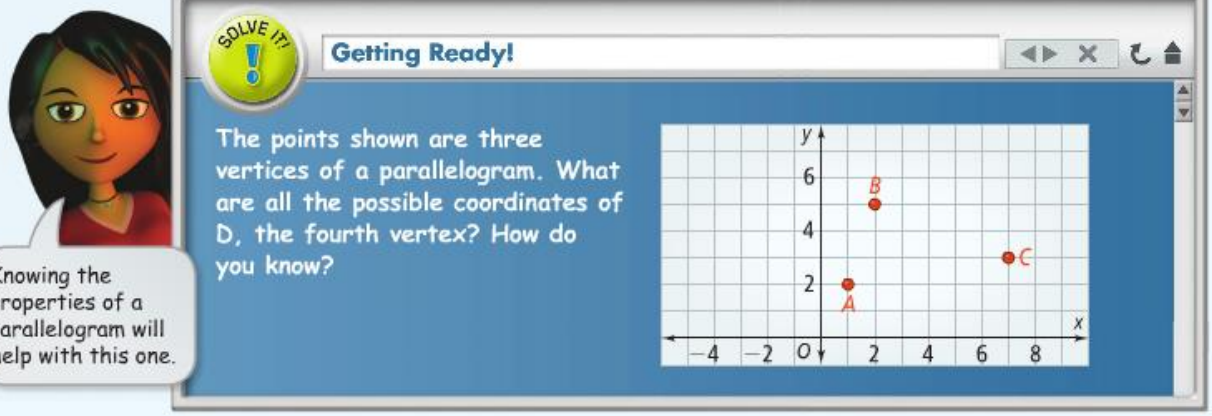
Find the slope of $\overline{XY}$.

See Lesson 3-7.

52. $X(0, a)$, $Y(-a, 2a)$ **53.** $X(-a, b)$, $Y(a, b)$ **54.** $X(a, 0)$, $Y(c + d, b)$

6-8 Applying Coordinate Geometry

Objective To name coordinates of special figures by using their properties



Lesson Vocabulary
- coordinate proof

In the Solve It, you found coordinates of a point and named it using numbers for the x- and y-coordinates. In this lesson, you will learn to use variables for the coordinates.

Essential Understanding You can use variables to name the coordinates of a figure. This allows you to show that relationships are true for a general case.

In Chapter 5, you learned about the segment joining the midpoints of two sides of a triangle. Here are three possible ways to place a triangle and its midsegment.

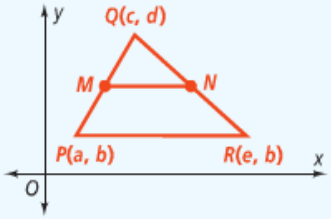


Figure 1

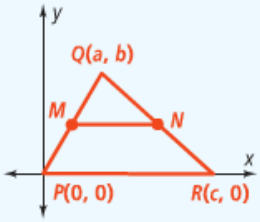


Figure 2

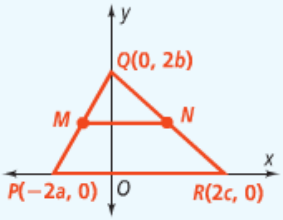


Figure 3

Figure 1 does not use the axes, so it requires more variables. Figures 2 and 3 have good placement. In Figure 2, the midpoint coordinates are $M\left(\frac{a}{2}, \frac{b}{2}\right)$ and $N\left(\frac{a+c}{2}, \frac{b}{2}\right)$. In Figure 3, the coordinates are $M(-a, b)$ and $N(c, b)$. You can see that Figure 3 is the easiest to work with.

To summarize, to place a figure in the coordinate plane, it is usually helpful to place at least one side on an axis or to center the figure at the origin. For the coordinates, try to anticipate what you will need to do in the problem. Then multiply the coordinates by the appropriate number to make your work easier.

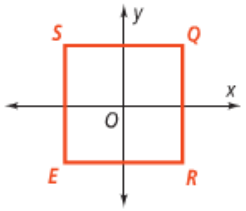
Problem 1 Naming Coordinates

Plan

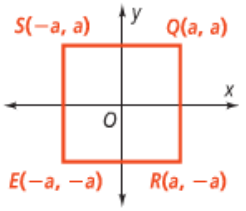
How do you start the problem?
Look at the position of the figure. Use the given information to determine how far each vertex is from the *x*- and *y*-axes.

What are the coordinates of the vertices of each figure?

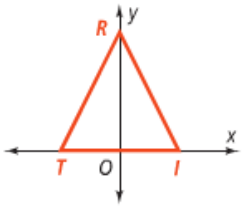
A ***SQRE* is a square where $SQ = 2a$. The axes bisect each side.**



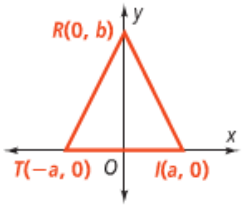
Since *SQRE* is a square centered at the origin and $SQ = 2a$, *S* and *Q* are each a units from each axis. The same is true for the other vertices.



B ***TRI* is an isosceles triangle where $TI = 2a$. The *y*-axis is a median.**

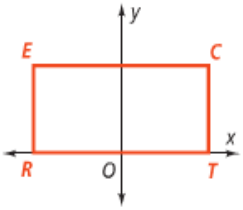


The *y*-axis is a median, so it bisects $\overline{TI}$. $TI = 2a$, so *T* and *I* are both a units from the *y*-axis. The height of *TRI* does not depend on a, so use a different variable for *R*.

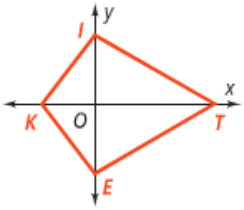


Got It? **1.** What are the coordinates of the vertices of each figure?

a. *RECT* is a rectangle with height a and length $2b$. The *y*-axis bisects $\overline{EC}$ and $\overline{RT}$.

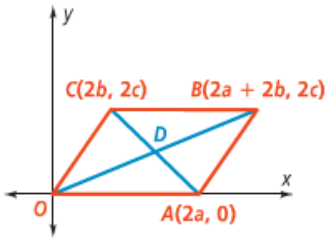


b. *KITE* is a kite where $IE = 2a$, $KO = b$, and $OT = c$. The *x*-axis bisects $\overline{IE}$.



Problem 2 Using Variable Coordinates

The diagram shows a general parallelogram with a vertex at the origin and one side along the x-axis. What are the coordinates of D, the point of intersection of the diagonals of $\square ABCO$? How do you know?



Know

- The coordinates of the vertices of $\square ABCO$
- $\overline{OB}$ bisects $\overline{AC}$ and $\overline{AC}$ bisects $\overline{OB}$

Need

The coordinates of D

Plan

Since the diagonals of a parallelogram bisect each other, the midpoint of each segment is their point of intersection. Use the Midpoint Formula to find the midpoint of one diagonal.

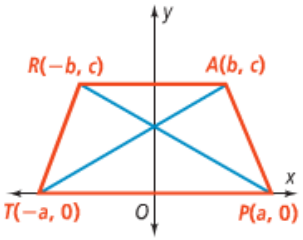
Use the Midpoint Formula to find the midpoint of $\overline{AC}$.

$$D = \text{midpoint of } \overline{AC} = \left(\frac{2a + 2b}{2}, \frac{0 + 2c}{2}\right) = (a + b, c)$$

The coordinates of the point of intersection of the diagonals of $\square ABCO$ are $(a + b, c)$.

Got It? **2. a.** **Reasoning** In Problem 2, explain why the x-coordinate of B is the sum of $2a$ and $2b$.

b. The diagram below shows a trapezoid with the base centered at the origin. Is the trapezoid isosceles? Explain.



You can use coordinate geometry and algebra to prove theorems in geometry. This kind of proof is called a **coordinate proof.** Sometimes it is easier to show that a theorem is true by using a coordinate proof rather than a standard deductive proof. It is useful to write a plan for a coordinate proof. Problem 3 shows you how.

Problem 3 Planning a Coordinate Proof

Plan

How do you start? Start by *drawing a diagram*. Think about how you want to place the figure in the coordinate plane.

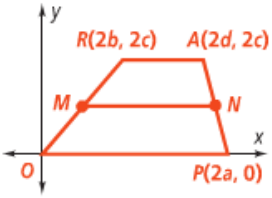
Plan a coordinate proof of the Trapezoid Midsegment Theorem (Theorem 6-21).

(1) The midsegment of a trapezoid is parallel to the bases.

(2) The length of the midsegment of a trapezoid is half the sum of the lengths of the bases.

Step 1 Draw and label a figure.

Midpoints will be involved, so use multiples of 2 to name coordinates.



Step 2 Write the *Given* and *Prove* statements.

Use the information on the diagram to write the statements.

Given: $\overline{MN}$ is the midsegment of trapezoid $ORAP$.

Prove: $\overline{MN} \parallel \overline{OP}$, $\overline{MN} \parallel \overline{RA}$, $MN = \frac{1}{2}(OP + RA)$

Step 3 Determine the formulas you will need. Then write the plan.

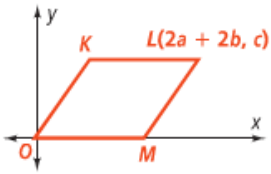
- First, use the Midpoint Formula to find the coordinates of M and N.
- Then, use the Slope Formula to determine whether the slopes of $\overline{MN}$, $\overline{OP}$, and $\overline{RA}$ are equal. If they are, $\overline{MN}$, $\overline{OP}$, and $\overline{RA}$ are parallel.
- Finally, use the Distance Formula to find and compare the lengths of $\overline{MN}$, $\overline{OP}$, and $\overline{RA}$.

Got It? **3.** Plan a coordinate proof of the Triangle Midsegment Theorem (Theorem 5-1).

Lesson Check

Do you know HOW?

Use the diagram at the right.



1. In $\square KLMO$, $OM = 2a$. What are the coordinates of K and M?
2. What are the slopes of the diagonals of $KLMN$?
3. What are the coordinates of the point of intersection of $\overline{KM}$ and $\overline{OL}$?

Do you UNDERSTAND?

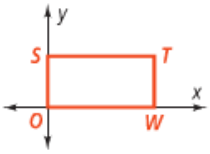
4. **Reasoning** How do variable coordinates generalize figures in the coordinate plane?
5. **Reasoning** A vertex of a quadrilateral has coordinates (a, b). The x-coordinates of the other three vertices are a or $-a$, and the y-coordinates are b or $-b$. What kind of quadrilateral is the figure?
6. **Error Analysis** A classmate says the endpoints of the midsegment of the trapezoid in Problem 3 are $\left(\frac{b}{2}, \frac{c}{2}\right)$ and $\left(\frac{d+a}{2}, \frac{c}{2}\right)$. What is your classmate's error? Explain.

Practice and Problem-Solving Exercises

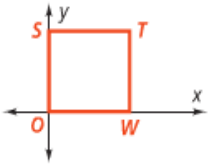
A Practice

Algebra What are the coordinates of the vertices of each figure? See Problem 1.

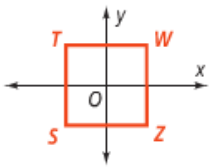
7. rectangle with base b and height h



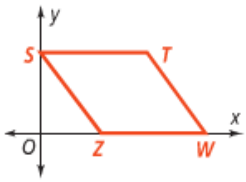
8. square with sides of length a



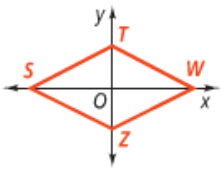
9. square centered at the origin, with side length b



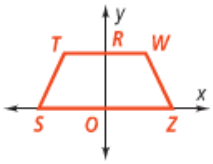
10. parallelogram where S is a units from the origin and Z is b units from the origin



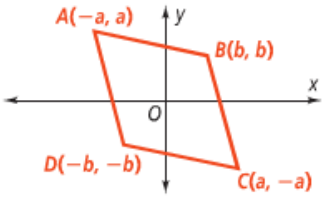
11. rhombus centered at the origin, with $SW = 2r$ and $TZ = 2t$



12. isosceles trapezoid with base centered at the origin, with base $2a$ and $OR = c$



13. The diagram below shows a parallelogram. Without using the Distance Formula, determine whether the parallelogram is a rhombus. How do you know? See Problem 2.



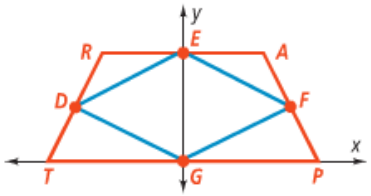
14. Plan a coordinate proof to show that the midpoints of the sides of an isosceles trapezoid form a rhombus. See Problem 3.

a. Name the coordinates of isosceles trapezoid $TRAP$ at the right, with bottom base length $4a$, top base length $4b$, and $EG = 2c$. The y-axis bisects the bases.

b. Write the *Given* and *Prove* statements.

c. How will you find the coordinates of the midpoints of each side?

d. How will you determine whether $DEFG$ is a rhombus?



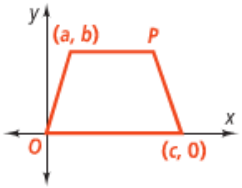
B Apply

15. Open-Ended Place a general quadrilateral in the coordinate plane.

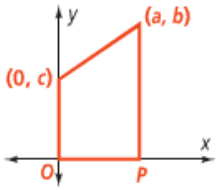
16. Reasoning A rectangle $LMNP$ is centered at the origin with $M(r, -s)$. What are the coordinates of P?

Give the coordinates for point P without using any new variables.

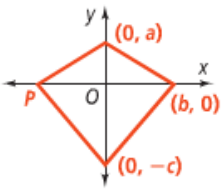
17. isosceles trapezoid



18. trapezoid with a right ∠



19. kite

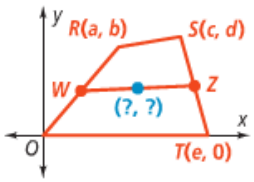


20. **a.** Draw a square whose diagonals of length $2b$ lie on the x- and y-axes.
b. Give the coordinates of the vertices of the square.
c. Compute the length of a side of the square.
d. Find the slopes of two adjacent sides of the square.
e. **Writing** Do the slopes show that the sides are perpendicular? Explain.

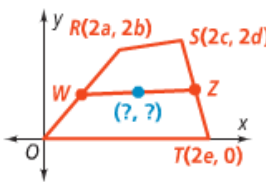
21. Make two drawings of an isosceles triangle with base length $2b$ and height $2c$.
a. In one drawing, place the base on the x-axis with a vertex at the origin.
b. In the second, place the base on the x-axis with its midpoint at the origin.
c. Find the lengths of the legs of the triangle as placed in part (a).
d. Find the lengths of the legs of the triangle as placed in part (b).
e. How do the results of parts (c) and (d) compare?

22. W and Z are the midpoints of $\overline{OR}$ and $\overline{ST}$, respectively. In parts (a)–(c), find the coordinates of W and Z.

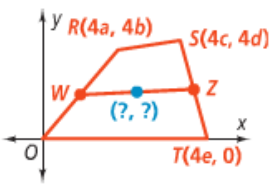
a.



b.



c.



d. You are to plan a coordinate proof involving the midpoint of $\overline{WZ}$. Which of the figures (a)–(c) would you prefer to use? Explain.

Plan the coordinate proof of each statement.

23. **Think About a Plan** The opposite sides of a parallelogram are congruent (Theorem 6-3).
- How will you place the parallelogram in a coordinate plane?
- What formulas will you need to use?

24. The diagonals of a rectangle bisect each other.

25. The consecutive sides of a square are perpendicular.

Classify each quadrilateral as precisely as possible.

26. $A(b, 2c)$, $B(4b, 3c)$, $C(5b, c)$, $D(2b, 0)$

27. $O(0, 0)$, $P(t, 2s)$, $Q(3t, 2s)$, $R(4t, 0)$

28. $E(a, b)$, $F(2a, 2b)$, $G(3a, b)$, $H(2a, -b)$

29. $O(0, 0)$, $L(-e, f)$, $M(f - e, f + e)$, $N(f, e)$

30. What property of a rhombus makes it convenient to place its diagonals on the x- and y-axes?

31. **Marine Archaeology** Marine archaeologists sometimes use a coordinate system on the ocean floor. They record the coordinates of points where artifacts are found. Assume that each diver searches a square area and can go no farther than b units from the starting point. Draw a model for the region one diver can search. Assign coordinates to the vertices without using any new variables.

Challenge

Here are coordinates for eight points in the coordinate plane ($q > p > 0$). $A(0, 0)$, $B(p, 0)$, $C(q, 0)$, $D(p + q, 0)$, $E(0, q)$, $F(p, q)$, $G(q, q)$, $H(p + q, q)$. Which four points, if any, are the vertices for each type of figure?

32. parallelogram

33. rhombus

34. rectangle

35. square

36. trapezoid

37. isosceles trapezoid

Standardized Test Prep

SAT/ACT

38. Which number of right angles is NOT possible for a quadrilateral to have?

(A) exactly one (B) exactly two (C) exactly three (D) exactly four

39. The vertices of a rhombus are located at $(a, 0)$, $(0, b)$, $(-a, 0)$, and $(0, -b)$, where $a > 0$ and $b > 0$. What is the midpoint of the side that is in Quadrant II?

(F) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (G) $\left(-\frac{a}{2}, \frac{b}{2}\right)$ (H) $\left(-\frac{a}{2}, -\frac{b}{2}\right)$ (I) $\left(\frac{a}{2}, -\frac{b}{2}\right)$

40. In $\square PQRS$, $PQ = 35$ cm and $QR = 12$ cm. What is the perimeter of $\square PQRS$?

(A) 23 cm (B) 47 cm (C) 94 cm (D) 420 cm

Short Response

41. In $\triangle PQR$, $PQ > PR > QR$. One angle measures 170. List all possible whole number values for $m\angle P$.

Mixed Review

42. Let $X(-2, 3)$, $Y(5, 5)$, and $Z(4, 10)$. Is $\triangle XYZ$ a right triangle? Explain. See Lesson 6-7.

Write (a) the inverse and (b) the contrapositive of each statement. See Lesson 2-2.

43. If $x = 51$, then $2x = 102$.

44. If $a = 5$, then $a^2 = 25$.

45. If $b < -4$, then b is negative.

46. If $c > 0$, then c is positive.

47. If the sum of the interior angle measures of a polygon is not 360, then the polygon is not a quadrilateral.

Get Ready! **To prepare for Lesson 6-9, do Exercises 48 and 49.** See Lesson 3-7.

48. Find the equation for the line that contains the origin and $(4, 5)$.

49. Find the equation for the line that contains (p, q) and has slope $\frac{a}{b}$.

Concept Byte

Use With Lesson 6-9

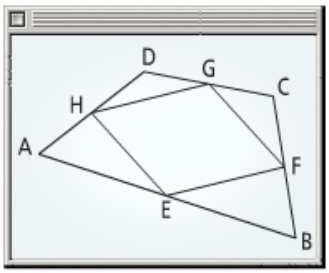
TECHNOLOGY

Quadrilaterals in Quadrilaterals

Activity

Construct

- Use geometry software to construct a quadrilateral *ABCD*.
- Construct the midpoint of each side of *ABCD*.
- Construct segments joining the midpoints, in order, to form quadrilateral *EFGH*.

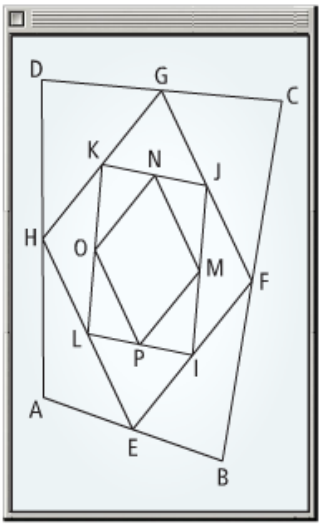


Investigate

- Measure the lengths of the sides of *EFGH* and their slopes.
- Measure the angles of *EFGH*.

What kind of quadrilateral does *EFGH* appear to be?

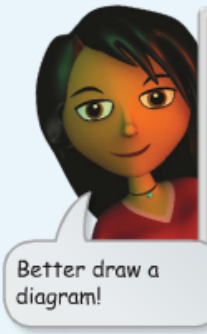
Exercises



1. Manipulate quadrilateral *ABCD*.
   a. Make a conjecture about the quadrilateral with vertices that are the midpoints of the sides of a quadrilateral.
   b. Does your conjecture hold when *ABCD* is concave?
   c. Can you manipulate *ABCD* so that your conjecture doesn't hold?
2. **Extend** Draw the diagonals of *ABCD*.
   a. Describe *EFGH* when the diagonals are perpendicular.
   b. Describe *EFGH* when the diagonals are congruent.
   c. Describe *EFGH* when the diagonals are both perpendicular and congruent.
3. Construct the midpoints of *EFGH* and use them to construct quadrilateral *IJKL*. Construct the midpoints of *IJKL* and use them to construct quadrilateral *MNOP*. For *MNOP* and *EFGH*, compare the ratios of the lengths of the sides, perimeters, and areas. How are the sides of *MNOP* and *EFGH* related?
4. **Writing** In Exercise 1, you made a conjecture as to the type of quadrilateral *EFGH* appears to be. Prove your conjecture. Include in your proof the Triangle Midsegment Theorem, "If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and half its length."
5. Describe the quadrilateral formed by joining the midpoints, in order, of the sides of each of the following. Justify each response.
   a. parallelogram b. rectangle c. rhombus d. square
   e. trapezoid f. isosceles trapezoid g. kite

6-9 Proofs Using Coordinate Geometry

Objective To prove theorems using figures in the coordinate plane



Getting Ready!

The coordinates of three vertices of a rectangle are $(-2a, 0)$, $(2a, 0)$, and $(2a, 2b)$. A diagonal joins one of these points with the fourth vertex. What are the coordinates of the midpoint of the diagonal? Justify your answer.

In the Solve It, the coordinates of the point include variables. In this lesson, you will use coordinates with variables to write a coordinate proof.

Essential Understanding You can prove geometric relationships using variable coordinates for figures in the coordinate plane.

Proof

Problem 1 Writing a Coordinate Proof

Use coordinate geometry to prove that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

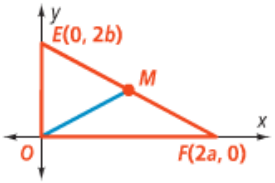
Plan

What formulas do you need?
You need to find the distance to a midpoint, so use the midpoint and distance formulas.

Given: $\triangle OEF$ is a right triangle.

M is the midpoint of $\overline{EF}$.

Prove: $EM = FM = OM$



Coordinate Proof:

By the Midpoint Formula, $M = \left(\frac{2a + 0}{2}, \frac{0 + 2b}{2}\right) = (a, b)$.

By the Distance Formula,

$OM = \sqrt{a^2 + b^2}$

$FM = \sqrt{(2a - a)^2 + (0 - b)^2}$
$= \sqrt{a^2 + b^2}$

$EM = \sqrt{(0 - a)^2 + (2b - b)^2}$
$= \sqrt{a^2 + b^2}$

Since $EM = FM = OM$, the midpoint of the hypotenuse is equidistant from the vertices of the right triangle.

Got It? 1. **Reasoning** What is the advantage of using coordinates $O(0, 0)$, $E(0, 2b)$, and $F(2a, 0)$ rather than $O(0, 0)$, $E(0, b)$, and $F(a, 0)$?

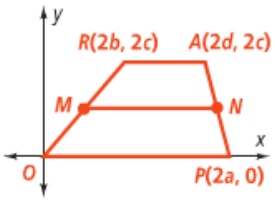
In Lesson 6-8, you wrote a plan for the proof of the Trapezoid Midsegment Theorem. Now you will write the full coordinate proof.

Plan

Refer to the plan from Lesson 6-8. Find the coordinates of M and N. Determine whether $\overline{MN}$ is parallel to $\overline{OP}$ and $\overline{RA}$. Then find and compare the lengths of $\overline{MN}$, $\overline{OP}$, and $\overline{RA}$.

Problem 2 Writing a Coordinate Proof

Write a coordinate proof of the Trapezoid Midsegment Theorem.



Given: $\overline{MN}$ is the midsegment of trapezoid $ORAP$.

Prove: $\overline{MN} \parallel \overline{OP}$, $\overline{MN} \parallel \overline{RA}$, $MN = \frac{1}{2}(OP + RA)$

Coordinate Proof:

Use the Midpoint Formula to find the coordinates of M and N.

$$M = \left(\frac{2b + 0}{2}, \frac{2c + 0}{2}\right) = (b, c)$$

$$N = \left(\frac{2a + 2d}{2}, \frac{0 + 2c}{2}\right) = (a + d, c)$$

Use the Slope Formula to determine whether $\overline{MN}$ is parallel to $\overline{OP}$ and $\overline{RA}$.

$$\text{slope of } \overline{MN} = \frac{c - c}{(a + d) - b} = 0$$

$$\text{slope of } \overline{RA} = \frac{2c - 2c}{2d - 2b} = 0$$

$$\text{slope of } \overline{OP} = \frac{0 - 0}{2a - 0} = 0$$

The three slopes are equal, so $\overline{MN} \parallel \overline{OP}$ and $\overline{MN} \parallel \overline{RA}$.

Use the Distance Formula to find and compare MN, OP, and RA.

$$MN = \sqrt{[(a + d) - b)]^2 + (c - c)^2} = a + d - b$$

$$OP = \sqrt{(2a - 0)^2 + (0 - 0)^2} = 2a$$

$$RA = \sqrt{(2d - 2b)^2 + (2c - 2c)^2} = 2d - 2b$$

$MN \stackrel{?}{=} \frac{1}{2}(OP + RA)$	Check that $MN = \frac{1}{2}(OP + RA)$ is true.
$a + d - b \stackrel{?}{=} \frac{1}{2}[2a + (2d - 2b)]$	Substitute.
$a + d - b = a + d - b$ ✓	Simplify.

So, (1) the midsegment of a trapezoid is parallel to its bases, and
(2) the length of the midsegment of a trapezoid is half the sum of the lengths of the bases.

Got It? **2.** Write a coordinate proof of the Triangle Midsegment Theorem (Theorem 5-1).

Lesson Check

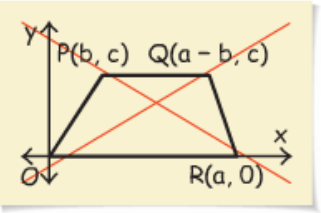
Do you know HOW?

1. Use coordinate geometry to prove that the diagonals of a rectangle are congruent.
   a. Place rectangle $PQRS$ in the coordinate plane with P at $(0, 0)$.
   b. What are the coordinates of Q, R, and S?
   c. Write the *Given* and *Prove* statements.
   d. Write a coordinate proof.

Do you UNDERSTAND?

2. **Reasoning** Describe a good strategy for placing the vertices of a rhombus for a coordinate proof.

3. **Error Analysis** Your classmate places a trapezoid on the coordinate plane. What is the error?



Practice and Problem-Solving Exercises

A Practice

Developing Proof **Complete the following coordinate proofs.**

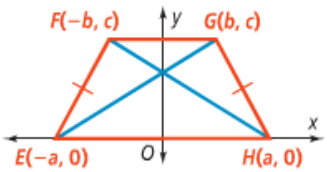
See Problems 1 and 2.

4. The diagonals of an isosceles trapezoid are congruent.

   Given: Trapezoid $EFGH$ with $\overline{EF} \cong \overline{GH}$

   Prove: $\overline{EG} \cong \overline{FH}$

   a. Find EG.
   b. Find FH.
   c. Explain why $\overline{EG} \cong \overline{FH}$.

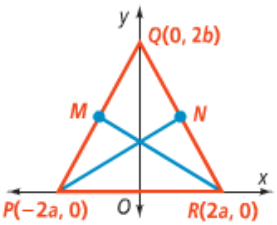


5. The medians drawn to the congruent sides of an isosceles triangle are congruent.

   Given: $\triangle PQR$ with $\overline{PQ} \cong \overline{RQ}$, M is the midpoint of $\overline{PQ}$, N is the midpoint of $\overline{RQ}$

   Prove: $\overline{PN} \cong \overline{RM}$

   a. What are the coordinates of M and N?
   b. What are PN and RM?
   c. Explain why $\overline{PN} \cong \overline{RM}$.



B Apply

Tell whether you can reach each type of conclusion below using coordinate methods. Give a reason for each answer.

6. $\overline{AB} \cong \overline{CD}$
7. $\overline{AB} \parallel \overline{CD}$
8. $\overline{AB} \perp \overline{CD}$
9. $\overline{AB}$ bisects $\overline{CD}$.
10. $\overline{AB}$ bisects $\angle CAD$.
11. $\angle A \cong \angle B$
12. $\angle A$ is a right angle.
13. $AB + BC = AC$
14. $\triangle ABC$ is isosceles.
15. Quadrilateral $ABCD$ is a rhombus.
16. $\overline{AB}$ and $\overline{CD}$ bisect each other.
17. $\angle A$ is the supplement of $\angle B$.
18. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are concurrent.

19. Flag Design The flag design at the right is made by connecting the midpoints of the sides of a rectangle. Use coordinate geometry to prove that the quadrilateral formed is a rhombus. Proof

20. Open-Ended Give an example of a statement that you think is easier to prove with a coordinate geometry proof than with a proof method that does not require coordinate geometry. Explain your choice.

Use coordinate geometry to prove each statement.

21. Think About a Plan If a parallelogram is a rhombus, its diagonals are perpendicular (Theorem 6-13). Proof

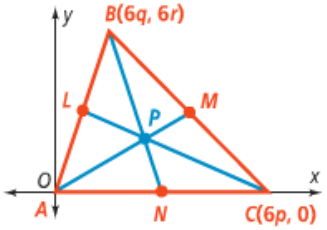
- How will you place the rhombus in a coordinate plane?
- What formulas will you need to use?

22. The altitude to the base of an isosceles triangle bisects the base.

23. If the midpoints of a trapezoid are joined to form a quadrilateral, then the quadrilateral is a parallelogram.

24. One diagonal of a kite divides the kite into two congruent triangles.

25. You learned in Theorem 5-8 that the centroid of a triangle is two thirds the distance from each vertex to the midpoint of the opposite side. Complete the steps to prove this theorem. Proof



a. Find the coordinates of points L, M, and N, the midpoints of the sides of $\triangle ABC$.

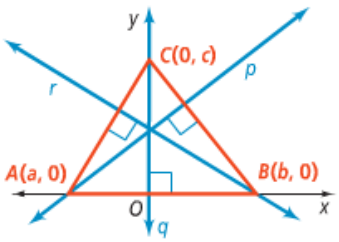
b. Find equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

c. Find the coordinates of point P, the intersection of $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$.

d. Show that point P is on $\overleftrightarrow{CL}$.

e. Use the Distance Formula to show that point P is two thirds the distance from each vertex to the midpoint of the opposite side.

26. Complete the steps to prove Theorem 5-9. You are given $\triangle ABC$ with altitudes p, q, and r. Show that p, q, and r intersect at a point (called the orthocenter of the triangle). Proof



a. The slope of $\overline{BC}$ is $\frac{c}{-b}$. What is the slope of line p?

b. Show that the equation of line p is $y = \frac{b}{c}(x - a)$.

c. What is the equation of line q?

d. Show that lines p and q intersect at $\left(0, \frac{-ab}{c}\right)$.

e. The slope of $\overline{AC}$ is $\frac{c}{-a}$. What is the slope of line r?

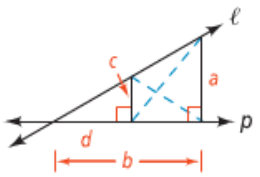
f. Show that the equation of line r is $y = \frac{a}{c}(x - b)$.

g. Show that lines r and q intersect at $\left(0, \frac{-ab}{c}\right)$.

h. What are the coordinates of the orthocenter of $\triangle ABC$?

Challenge

27. Multiple Representations Use the diagram at the right.



a. Explain using area why $\frac{1}{2}ad = \frac{1}{2}bc$ and therefore $ad = bc$.

b. Find two ratios for the slope of ℓ. Use these two ratios to show that $ad = bc$.

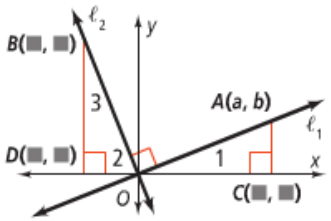
28. Prove: If two lines are perpendicular, the product of their slopes is -1.

Proof

a. Two nonvertical lines, ℓ_1 and ℓ_2, intersect as shown at the right. Find the coordinates of C.

b. Choose coordinates for D and B. (*Hint:* Find the relationship between $\angle 1$, $\angle 2$, and $\angle 3$. Then use congruent triangles.)

c. Complete the proof that the product of slopes is -1.



Standardized Test Prep

GRIDDED RESPONSE

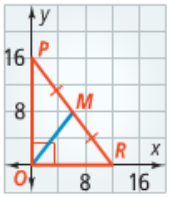
SAT/ACT

29. The endpoint of a segment is $(27, -3)$. The midpoint is $(3, 4)$. What is the length of the segment?

30. In the diagram of $\triangle POR$ at the right, $PO = 16$ and $OR = 12$. What is OM?

31. $\square FGHI$ has sides with lengths $FG = 2x + 5$, $GH = x + 7$, $HI = 3x - 2$, and $FI = 2x$. What is the length of the longer sides of $\square FGHI$?

32. In $\triangle ABC$, $m\angle A = 55$. If $m\angle C$ is twice $m\angle A$, what is $m\angle B$?



Mixed Review

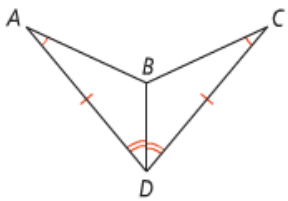
33. A rectangle $ABCD$ is centered at the origin with $A(-a, b)$. Without using any new variables, what are the coordinates of point C?

See Lesson 6-8.

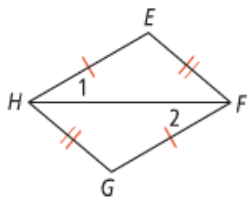
Explain how you can use SSS, SAS, ASA, or AAS with corresponding parts of congruent triangles to prove each statement true.

See Lessons 4-2, 4-3, and 4-4.

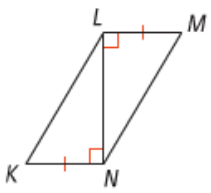
34. $\overline{AB} \cong \overline{CB}$



35. $\angle 1 \cong \angle 2$



36. $\angle K = \angle M$



Get Ready! **To prepare for Lesson 7-1, do Exercises 37–40.**

Algebra Solve. Round to the nearest tenth if necessary.

See p. 827.

37. $x^2 = 144$ **38.** $r^2 - 3 = 61$ **39.** $y^2 + 10 = 35$ **40.** $7^2 + k^2 = 18^2$

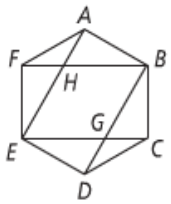
6 Pull It All Together

BIG idea Measurement and Reasoning and Proof

You can find the interior angle measures in a regular polygon and then use what you know about triangles and special quadrilaterals to analyze complex figures.

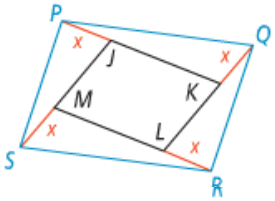
Task 1

ABCDEF is a regular hexagon. What is the most precise classification of quadrilateral *GBHE*? How do you know? What are the interior angle measures of *GBHE*?



Task 2

JKLM is a parallelogram. If you extend each side by a distance x, what kind of quadrilateral is *PQRS*? How do you know?

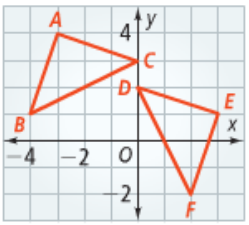


BIG idea Coordinate Geometry and Reasoning and Proof

You can use coordinates and certain relationships between pairs of corresponding parts to prove triangles congruent in the coordinate plane.

Task 3

What are two methods for proving the two triangles congruent? Use one of your methods along with coordinate geometry to prove that the two triangles are congruent.



6 Chapter Review

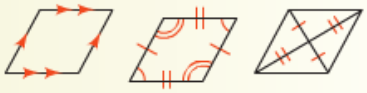
Connecting BIG ideas and Answering the Essential Questions

1 Measurement
You can find the sum of the interior angle measures of any polygon using a formula based on its number of sides.

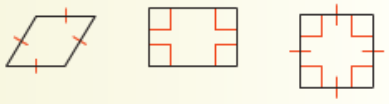
Polygon Angle-Sum Theorems (Lesson 6-1)
Sum $= (n - 2)180$, where n is the number of sides

2 Reasoning and Proof
If you know certain information about the sides, angles, or diagonals of a quadrilateral, you can classify it.

Parallelograms (Lessons 6-2 and 6-3)



Special Parallelograms (Lessons 6-4 and 6-5)



Trapezoids and Kites (Lesson 6-6)

3 Coordinate Geometry
Coordinate proofs use variable coordinates to prove relationships in the coordinate plane.

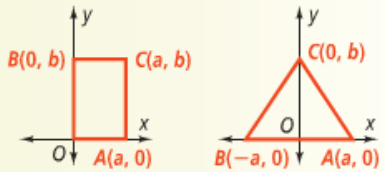
Applying Coordinate Geometry (Lessons 6-7 and 6-8)

midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

slope: $\frac{y_2 - y_1}{x_2 - x_1}$

Coordinate Proofs (Lesson 6-9)



Chapter Vocabulary

- base, base angle, and leg of a trapezoid (p. 389)
- consecutive angles (p. 360)
- coordinate proof (p. 408)
- equiangular, equilateral polygon (p. 354)
- isosceles trapezoid (p. 389)
- kite (p. 392)
- midsegment of a trapezoid (p. 391)
- opposite angles (p. 359)
- opposite sides (p. 359)
- parallelogram (p. 359)
- rectangle (p. 375)
- regular polygon (p. 354)
- rhombus (p. 375)
- square (p. 375)
- trapezoid (p. 389)

Choose the vocabulary term that correctly completes the sentence.

1. A parallelogram with four congruent sides is a(n) ___?___.

2. A polygon with all angles congruent is a(n) ___?___.

3. Angles of a polygon that share a side are ___?___.

4. A(n) ___?___ is a quadrilateral with exactly one pair of parallel sides.

6-1 The Polygon Angle-Sum Theorems

Quick Review

The sum of the measures of the interior angles of an n-gon is $(n - 2)180$. The measure of one interior angle of a regular n-gon is $\frac{(n - 2)180}{n}$. The sum of the measure of the exterior angles of a polygon, one at each vertex, is 360.

Example

Find the measure of an interior angle of a regular 20-gon.

$$\text{Measure} = \frac{(n - 2)\,180}{n} \quad \text{Corollary to the Polygon Angle-Sum Theorem}$$

$$= \frac{(20 - 2)180}{20} \quad \text{Substitute.}$$

$$= \frac{18 \cdot 180}{20} \quad \text{Simplify.}$$

$$= 162$$

The measure of an interior angle is 162.

Exercises

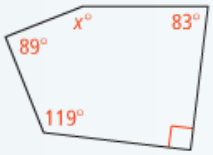
Find the measure of an interior angle and an exterior angle of each regular polygon.

5. hexagon **6.** 16-gon **7.** pentagon

8. What is the sum of the exterior angles for each polygon in Exercises 5–7?

Find the measure of the missing angle.

9.



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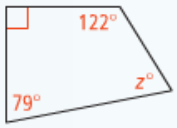


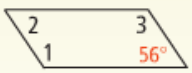
6-2 Properties of Parallelograms

Quick Review

Opposite sides and **opposite angles** of a **parallelogram** are congruent. **Consecutive angles** in a parallelogram are supplementary. The diagonals of a parallelogram bisect each other. If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example

Find the measures of the numbered angles in the parallelogram.

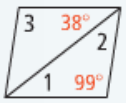


Since consecutive angles are supplementary, $m\angle 1 = 180 - 56$, or 124. Since opposite angles are congruent, $m\angle 2 = 56$ and $m\angle 3 = 124$.

Exercises

Find the measures of the numbered angles for each parallelogram.

11.



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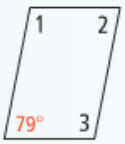

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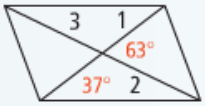


14.

Find the values of x and y in $\square ABCD$.

15. $AB = 2y, BC = y + 3, CD = 5x - 1, DA = 2x + 4$

16. $AB = 2y + 1, BC = y + 1, CD = 7x - 3, DA = 3x$

6-3 Proving That a Quadrilateral Is a Parallelogram

Quick Review

A quadrilateral is a parallelogram if any one of the following is true.

- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are congruent.
- Consecutive angles are supplementary.
- Both pairs of opposite angles are congruent.
- The diagonals bisect each other.
- One pair of opposite sides is both congruent and parallel.

Example

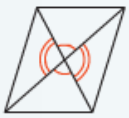
Must the quadrilateral be a parallelogram?

Yes, both pairs of opposite angles are congruent.

Exercises

Determine whether the quadrilateral must be a parallelogram.

17.



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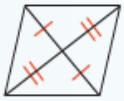
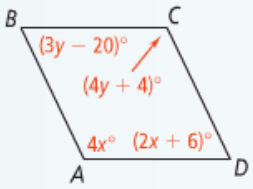
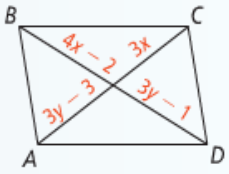


Algebra **Find the values of the variables for which *ABCD* must be a parallelogram.**

19.



20.



6-4 Properties of Rhombuses, Rectangles, and Squares

Quick Review

A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.

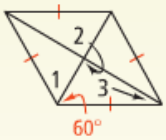
A **square** is a parallelogram with four congruent sides and four right angles.

The diagonals of a rhombus are perpendicular. Each diagonal bisects a pair of opposite angles.

The diagonals of a rectangle are congruent.

Example

What are the measures of the numbered angles in the rhombus?

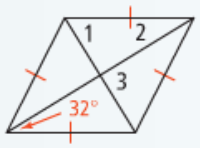


$m\angle 1 = 60$	Each diagonal of a rhombus bisects a pair of opposite angles.
$m\angle 2 = 90$	The diagonals of a rhombus are $\perp$.
$60 + m\angle 2 + m\angle 3 = 180$	Triangle Angle-Sum Thm.
$60 + 90 + m\angle 3 = 180$	Substitute.
$m\angle 3 = 30$	Simplify.

Exercises

Find the measures of the numbered angles in each special parallelogram.

21.



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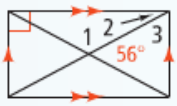


Determine whether each statement is *always*, *sometimes*, or *never* true.

23. A rhombus is a square.

24. A square is a rectangle.

25. A rhombus is a rectangle.

26. The diagonals of a parallelogram are perpendicular.

27. The diagonals of a parallelogram are congruent.

28. Opposite angles of a parallelogram are congruent.

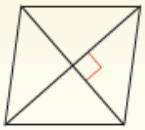
6-5 Conditions for Rhombuses, Rectangles, and Squares

Quick Review

If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Example

Can you conclude that the parallelogram is a rhombus, rectangle, or square? Explain.

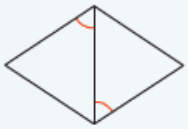


Yes, the diagonals are perpendicular, so the parallelogram is a rhombus.

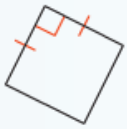
Exercises

Can you conclude that the parallelogram is a rhombus, rectangle, or square? Explain.

29.

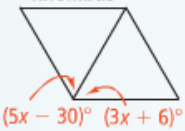


30.

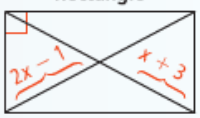


For what value of x is the figure the given parallelogram? Justify your answer.

31. Rhombus



32. Rectangle



6-6 Trapezoids and Kites

Quick Review

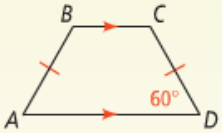
The parallel sides of a **trapezoid** are its **bases** and the nonparallel sides are its **legs.** Two angles that share a base of a trapezoid are **base angles** of the trapezoid. The **midsegment of a trapezoid** joins the midpoints of its legs.

The base angles of an isosceles trapezoid are congruent. The diagonals of an isosceles trapezoid are congruent.

The diagonals of a kite are perpendicular.

Example

***ABCD* is an isosceles trapezoid. What is $m\angle C$?**



Since $\overline{BC} \parallel \overline{AD}$, $\angle C$ and $\angle D$ are same-side interior angles.

$m\angle C + m\angle D = 180$ Same-side interior angles are supplementary.

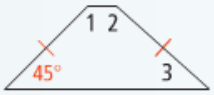
$m\angle C + 60 = 180$ Substitute.

$m\angle C = 120$ Subtract 60 from each side.

Exercises

Find the measures of the numbered angles in each isosceles trapezoid.

33.



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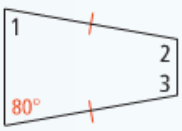
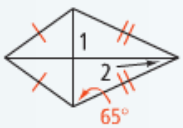
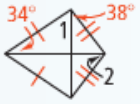


Find the measures of the numbered angles in each kite.

35.



36.



37. Algebra A trapezoid has base lengths of $(6x - 1)$ units and 3 units. Its midsegment has a length of $(5x - 3)$ units. What is the value of x?

6-7 Polygons in the Coordinate Plane

Quick Review

To determine whether sides or diagonals are congruent, use the Distance Formula. To determine the coordinate of the midpoint of a side, or whether the diagonals bisect each other, use the Midpoint Formula. To determine whether opposite sides are parallel, or whether diagonals or sides are perpendicular, use the Slope Formula.

Example

$\triangle XYZ$ has vertices $X(1, 0)$, $Y(-2, -4)$, and $Z(4, -4)$. Is $\triangle XYZ$ *scalene, isosceles,* or *equilateral*?

To find the lengths of the legs, use the Distance Formula.

$$XY = \sqrt{(-2 - 1)^2 + (-4 - 0)^2} = \sqrt{9 + 16} = 5$$

$$YZ = \sqrt{(4 - (-2))^2 + (-4 - (-4))^2} = \sqrt{36 + 0} = 6$$

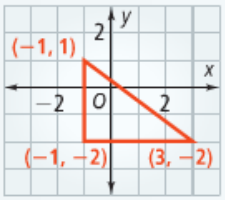
$$XZ = \sqrt{(4 - 1)^2 + (-4 - 0)^2} = \sqrt{9 + 16} = 5$$

Two side lengths are equal, so $\triangle XYZ$ is isosceles.

Exercises

Determine whether $\triangle ABC$ is *scalene, isosceles,* or *equilateral.*

38.



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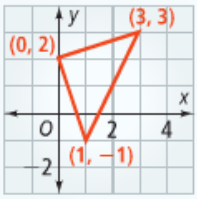


What is the most precise classification of the quadrilateral?

40. $G(2, 5)$, $R(5, 8)$, $A(-2, 12)$, $D(-5, 9)$

41. $F(-13, 7)$, $I(1, 12)$, $N(15, 7)$, $E(1, -5)$

42. $Q(4, 5)$, $U(12, 14)$, $A(20, 5)$, $D(12, -4)$

43. $W(-11, 4)$, $H(-9, 10)$, $A(2, 10)$, $T(4, 4)$

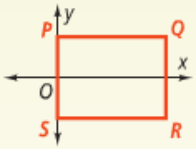
6-8 and 6-9 Coordinate Geometry and Coordinate Proofs

Quick Review

When placing a figure in the coordinate plane, it is usually helpful to place at least one side on an axis. Use variables when naming the coordinates of a figure in order to show that relationships are true for a general case.

Example

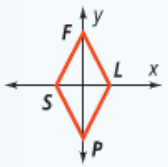
Rectangle $PQRS$ has length a and width $4b$. The x-axis bisects $\overline{PS}$ and $\overline{QR}$. What are the coordinates of the vertices?



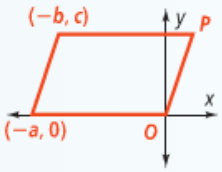
Since the width of $PQRS$ is $4b$ and the x-axis bisects $\overline{PS}$ and $\overline{QR}$, all the vertices are $2b$ units from the x-axis. $\overline{PS}$ is on the y-axis, so $P = (0, 2b)$ and $S = (0, -2b)$. The length of $PQRS$ is a, so $Q = (a, 2b)$ and $R = (a, -2b)$.

Exercises

44. In rhombus $FLPS$, the axes form the diagonals. If $SL = 2a$ and $FP = 4b$, what are the coordinates of the vertices?



45. The figure at the right is a parallelogram. Give the coordinates of point P without using any new variables.



46. Use coordinate geometry to prove that the quadrilateral formed by connecting the midpoints of a kite is a rectangle.

6 Chapter Test

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Do you know HOW?

1. What is the sum of the interior angle measures of a polygon with 15 sides?

2. What is the measure of an exterior angle of a 25-gon?

Graph each quadrilateral *ABCD*. Then determine the most precise name for it.

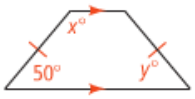
3. $A(1, 2), B(11, 2), C(7, 5), D(4, 5)$

4. $A(3, -2), B(5, 4), C(3, 6), D(1, 4)$

5. $A(1, -4), B(1, 1), C(-2, 2), D(-2, -3)$

Algebra Find the values of the variables for each quadrilateral.

6.



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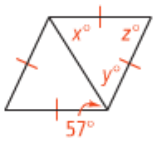

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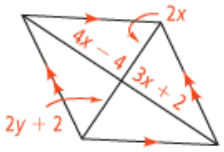

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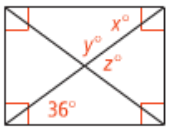

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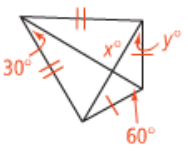

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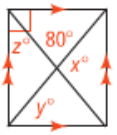
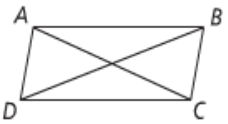


Does the information help you prove that *ABCD* is a parallelogram? Explain.



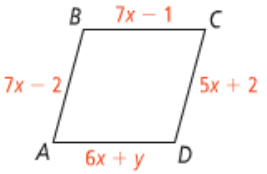
12. $\overline{AC}$ bisects $\overline{BD}$.

13. $\overline{AB} \cong \overline{DC}, \overline{AB} \parallel \overline{DC}$

14. $\overline{AB} \cong \overline{DC}, \overline{BC} \cong \overline{AD}$

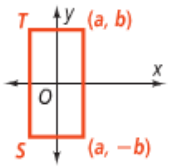
15. $\angle DAB \cong \angle BCD, \angle ABC \cong \angle CDA$

16. **Algebra** Determine the values of the variables for which *ABCD* is a parallelogram.

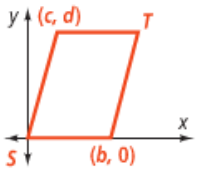


Give the coordinates for points *S* and *T* without using any new variables. Then find the midpoint and the slope of $\overline{ST}$.

17. rectangle

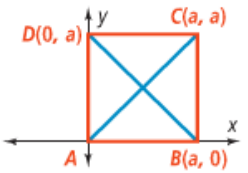


18. parallelogram



19. Prove that the diagonals of square *ABCD* are congruent.

20. Sketch two noncongruent parallelograms *ABCD* and *EFGH* such that $\overline{AC} \cong \overline{BD} \cong \overline{EG} \cong \overline{FH}$.



Do you UNDERSTAND?

21. **Open-Ended** Write the coordinates of four points that determine each figure with the given conditions. One vertex is at the origin and one side is 3 units long.
   a. square **b.** parallelogram
   c. rectangle **d.** trapezoid

22. **Writing** Explain why a square cannot be a kite.

23. **Error Analysis** Your classmate says, "If the diagonals of a quadrilateral intersect to form four congruent triangles, then the quadrilateral is always a parallelogram." How would you correct this statement? Explain your answer.

24. **Reasoning** *PQRS* has vertices $P(0, 0)$, $Q(4, 2)$, and $S(4, -2)$. Its diagonals intersect at $H(4, 0)$. What are the possible coordinates of *R* for *PQRS* to be a kite? Explain.